Reputation effects in regulatory enforcement

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Abstract

We show that, under plausible hypotheses, an enforcement agency without commitment power will not undertake any enforcement effort at all in a static game. Indeed, punishment of noncompliant agents does not necessarily bring social benefits in itself. In a dynamic framework, however, the enforcement agency might inspect private agents in order to develop a reputation that it will inspect in the future. However small the private agents' prior beliefs that they will be inspected, the agency can obtain almost perfect compliance if the game lasts long enough. Our model with reputation effects does however not converge to a model with perfect commitment.

1 Introduction

As any kind of regulation imposes costs on private agents, we should not expect compliance with regulations unless the government enforces them. This means that any analysis of regulatory policy is incomplete as long as enforcement issues are not taken into account.

The economic analysis of criminal law provides a good framework to incorporate enforcement issues into regulatory economics. It exceeds the scope of this contribution to review this literature - for a recent survey, see Polinsky and Shavell (2000).

There is one important problem however that has largely been ignored until now. Indeed, it is commonly assumed that the enforcer commits a given amount of resources to the detection of offenders, and that these resources determine directly the probability of detection, even if these probabilities are not an optimal reaction to the behavior of the potential offenders[1]. In other words, the enforcement agency is implicitly assumed to have full commitment power with respect to its policies.

Few authors seems thus to recognize the implicit assumption in most models that an enforcement agency can announce a detection probability and subsequently stick to it. There are some exceptions, though (see, for instance, Grieson and Singh (1990), Holler (1993), Reinganum and Wilde (1986), Saha and Poole (2000) and Tsebelis (1990a, 1990b,1993)).

There are however several reasons to doubt the plausibility of this commitment hypotheses. If the legislator has delegated some policy decisions to the enforcement agency, the agency can deviate from announced policies if sticking to these policies is not optimal *ex post* from her point of view (and, as Reinganum and Wilde (1986) have pointed out, it is the agency that moves last). It is then not reasonable to assume that the agency has full commitment power[2]. Moreover , as potential offenders cannot observe the probability of inspection, it is impossible to verify if an announced policy has really been carried out (see Reinganum and Wilde (1986) and Melumad and Mookherjee (1989)).

In a series of challenging papers, Tsebelis (1990a,1990b,1993) has argued that dropping the commitment hypothesis has dramatic effects on the deterrence effect of higher penalties: higher penalties would not lead to lower crime, but to lower crime enforcement. In equilibrium, the level of crime in society is then independent from penalties.

Although Tseblis has considered several extensions of the basic model, we think that there is at least one more aspect of law enforcement that has been neglected until now. Indeed, in this paper, we show that Tsebelis's strong results depend on the assumption that the enforcement agency does not try to build a reputation for tough law enforcement. Although the fines do not affect equilibrium *behavior* directly, they do determine the *type* of equilibrium that will emerge.

The general setting of our model is the following. We consider a private agent, who can choose between complying and not complying with a regulation. The enforcement agency can only detect noncompliance if it inspects the agent, but if it does inspect the agent, it can detect noncompliance with certainty. This is a reasonable first approximation of several enforcement problems. We could think for instance of environmental enforcement, tax-auditing, enforcement of safety regulations, etc. On the other hand, the analysis that follows does not apply to the analysis of crime such as murder or theft, where the transgression of the law can be observed without inspections, but where the identity of the criminal is not always certain.

In Section 2, we illustrate the importance of the commitment hypothesis by showing that, under certain plausible hypotheses, there is no reason to believe that an enforcement agency without commitment power would undertake any enforcement effort at all in a static game. In a dynamic framework, however, the enforcement agency might inspect private agents in order to develop a reputation that they will be inspected in the future. If the private agents believe this, the agency can obtain some compliance without commitment power. The basic structure of such a dynamic game is developed in Section 3. In Section 4, we confirm the standard result that the stage game equilibrium is also maintained in the repeated interaction (even though this repeated interaction is not a repeated game). In Section 5, we show that, if inspection costs are too high compared to the social benefits of compliance, nothing changes compared to the static game. In Section 6, however, we show that if inspection costs are low enough, the agency can obtain perfect compliance in a dynamic framework as long as the private agents' *a priori* belief that they will be inspected *and* the fine for noncompliance are high enough compared to private compliance costs. If the *a priori* belief and the fine are not high enough, we show in Sections 7 and 8 that the agency can also obtain perfect compliance in "long" games, except at the end of the repeated interaction - again, the fine determines the round where perfect compliance stops. In Section 9 we show that, even in the long run, our model with reputation effects does not provide a justification for the commitment hypothesis. Concluding remarks are formulated in Section 10.

It is important to realize that the results we shall obtain are not trivial extensions of generally known results in the theory of reputation effects. For instance, Kreps and Wilson (1982), Milgrom and Roberts (1982) and Fudenberg and Levine (1989) consider reputation effects where a single long-run player faces a sequence of opponents. These opponents only play once. The long-run player is however able to build a reputation because the short-run players can observe play in the previous rounds. In our model, the enforcement agency meets the same private agent over and over again. Our approach is closer to the model developed by Fudenberg and Kreps (1987), who consider a single "big" player who faces a large number of long-lived opponents. However, their results do not generalize to the problem we are looking at.

2 The static game

We first consider a very simple static game *without* repeated interactions.[3]

We shall assume that we have only one private agent, who is subject to a regulation. Complying with this regulation costs α.

Inspecting the private agent costs b. If a private agent is inspected and is found in noncompliance (the probability of inspection and of detection are thus equal in our model), then it will have to pay a fixed fine $\Psi > 0$ with certainty. This fine is exogenous.

The private agent can choose between complying and not complying, and the agency can choose between inspecting and not inspecting.

We assume that, for the agency, the cost of compliance is D_c; the cost of noncompliance will be represented as D_{nc}. Note that D_{nc} and D_c can be given a wide range of interpretations. For instance, if the agency maximizes social welfare, D_{nc} and D_c are the external costs (net of private compliance costs) of the private agent's actions. We assume that $D_{nc} > D_c$: otherwise, the agency would have no reason to pursue compliance.

We also assume that the agency derives *some* benefit $\triangle$ from inspecting a *noncompliant private agent*. For instance, the career perspectives of the agency's staff may depend on the number of detected noncompliant private agents, or the staff may derive some moral satisfaction from fining noncompliant private agents. Alternatively, the courts might have the authority to put a noncompliant private agent in compliance; $\triangle$ then represents the external benefit (net of private compliance costs) of inspecting a noncompliant private agent. In order to allow for this latter interpretation, we shall from now on assume that a private agent that is found in noncompliance has to incur a fraction θ of the costs of purchasing the new abatement technology, where $\theta \in \{0, 1\}$. However, we shall assume that there is no redistribution of fines to the agency, so that $\triangle$ is completely independent from Ψ. We assume that $\Psi > (1 - \theta)\alpha$: otherwise, the private agent would never comply, even if he is inspected with certainty. Clearly, it does not make sense to analyze such a case.

The assumption that the enforcement agency cannot commit means that the enforcement agency cannot act as a Stackelberg leader with respect to inspection probabilities. Instead, we shall use the Nash equilibrium concept: the private agent's strategy must be the best reaction to the enforcement agency's strategy, but the enforcement agency's strategy must also be the best response to the private agent's strategy.

The payoff-matrix for this game are represented in Table 1.

	The private agent complies	The private agent does not comply
The agency inspects the private agent	$-b - D_c, -\alpha$	$-b - D_{nc} + \triangle, -\Psi - \theta\alpha$
The agency does not inspect the private agent	$-D_c, -\alpha$	$-D_{nc}, 0$

Table 1: Payoff-matrix

Compare this Table with Table 1 in Tsebelis (1990,a). It can be easily verified that the only difference between this payoff-structure and the payoff-structure used by Tsebelis, is that Tsebelis assumes *a priori* that the enforcement agency prefers to inspect the private agent if the private agent does not comply. Under this assumption, Tsebelis obtains equilibria in mixed strategies - see the original paper for a thorough discussion.

Now consider the possibility that $b > \triangle$.

One possible interpretation of this case is the following (for an alternative interpretation, see Tsebelis (1990 a p. 275)). Suppose that the court does not have the power to put a noncompliant private agent in compliance. If the agency only cares about the external effects, then inspecting the private agent brings no benefits in itself and $\triangle \leq 0$.

From the payoff-matrix (see Table 1), inspecting a private agent is then a dominated strategy. After elimination of this dominated strategy, we see that the optimal choice for the private agent is not to comply.

Proposition 2.1 *If $b > \triangle$, the only strategy-pair that survives iterated elimination of strictly dominated strategies in a non-repeated interaction is: (the agency does not inspect the private agent, the private agent does not comply).*

Inspecting thus only makes sense if we assume that the agency can somehow change the private agent's behavior, either in the short or in the long run.

The literature on long-run relationships suggests indeed that other equilibria are possible if the enforcement agency and the private agents meet more than once. Basically, two modeling approaches are popular in the literature.

A first possibility is to develop a model with reputation effects. As Fudenberg and Tirole (1995: 367) put it:

> (...) a player who plays the same game repeatedly may try to develop a reputation for certain kinds of play. The idea is that if the player always plays in the same way, his opponents will come to expect him to play that way in the future and will adjust their own play accordingly. The question is then when and whether a player will be able to develop or maintain the reputation he desires (...) To model the possibility that players are concerned about their reputation, we suppose that there is incomplete information about each player's type, with different types expected to play in different ways. Each player's reputation is then summarized by his opponents' current beliefs about his type.

A second possibility is to consider infinitely repeated games with complete information. This problem however exceeds the scope of this paper.

Indeed, any modeling of repeated games would be far from trivial. Most results in the theory of repeated games are based on the assumption that the actions of all players are revealed after each interaction (see, for instance, Fudenberg and Tirole (1995: 146-147). However, unless an inspection takes place, the action chosen by the private agent in any stage of the game will never be revealed to the enforcement agency (during the period of the game). And, as

Mertens (1990: 80) has stated: "Claiming utilities (i.e., essentially the actual outcomes) to be known is essentially negating the whole quality-control problem, which is an essential part of most principal-agent relationships, if not the single most important (...)".

The approach we have chosen here is to develop a model with reputation effects.

3 Setting of the dynamic model

We consider a game between an enforcement agency and a private agent. Unless stated explictly otherwise, we keep all the assumptions and notations of Section 2.

We assume here that the private agent is not certain with respect to the enforcement's agency's payoff structure. More specifically, it believes that the agency can take two types:

- Type I always inspects the private agent. A possible cause for this kind of behavior is that the Type I agency has bureaucratic objectives (for instance, promotions depend only on the number of inspections or on the amount of fines collected) and that it does not care about inspection costs (as it receives a fixed budget every year anyway). Another possibility is that the inspectors draw moral satisfaction from fining noncompliant private agents (for an alternative interpretation, see Tsebelis (1990 a p. 275)).
- Type II never inspects the private agents in a not-repeated interaction because $b > \triangle$ (see Proposition 2.1).

This means that we do not consider the possibility that the private agents believes that $\triangle > b$. As we have already mentioned above, Tsebelis (1990 a p. 275)) has shown in a formally equivalent model that the agency and the private agent will then both play mixed strategies in the stage game. Adding this possibility would add a lot in technical complexity without providing much additional insights.

The private agent's a-priori belief that the agency is type I is q_0.

The purpose of this paper is to show how the *agency* can develop a reputation that it always inspects the private agent even if it would not do so in a static setting. Therefore, we only consider the optimal strategy for a type II agency and we assume that the agency is certain with respect to the private agent's objectives.

We consider a game with a finite number of interactions. n shall be used to represent the number of interactions. In each stage, the private agent can choose between complying and not complying and the agency can choose between inspecting and not inspecting.

In a repeated interaction, the players' actions can influence future payoffs in two ways:

- They can have a physical impact on future payoffs. Here we shall assume that the actions do not bind the players physically in the next stage. More specifically, if compliance requires the purchase of equipment, we assume that this equipment fully depreciates.

- Players can condition their play on the behavior they have observed in the past rounds. To simplify matters, we shall assume that the enforcement agency does not condition its play on the private agent's behavior in the previous stages. The private agent can then act myopically: it does not have to take into account how its actions will affect the agency's future behavior. This means of course that we exclude a whole possible range of equilibria. This is not because these equilibria are less interesting than the ones we will consider here. However, the emphasis here is on why the agency would inspect the private agent in repeated interaction even if it never inspects the private agent in a one-shot game. To obtain such results, it is enough to assume that the private agent looks at the agency's past behavior. We shall thus stick to the simplest possible framework.

Enforcement agency and private agent face the same discount rate δ.

We work through backwards induction. Unless stated explicitly otherwise, we shall use the perfect Bayesian equilibrium (PBE) as solution concept. This means that the private agent's strategy must be optimal, *given the agency's strategy*, but the agency's strategy must also be optimal, *given the private agent's strategy.* Moreover, the private agent's beliefs with respect to the agency's type must be obtained from the agency's equilibrium strategy and from the observation of its observed actions, using Bayes' rule (for a formal treatment of this concept, see for instance Fudenberg and Tirole (1995: 325-326)).

This implies that before proceeding further, we need to know how the private agent should form its beliefs with respect to the agency's type.

In the first round, these beliefs are just the a-priori probabilities.

Note now that, in the last round, a type I agency always inspects the private agent and a type II agency never inspects the private agent. Moreover, as soon as the private agent has once not been inspected, it knows that the agency is type II (because the private agent knows that the type I agency always inspects) and the private agent will never comply in the last round (as it will never be inspected in the last round). If the private agent does not comply in round n, then the agency's expected future costs in round $n-1$ are (where $P_{t\alpha}$ is the probability that the private agent complies in round t and P_t is the probability that the agency inspects the private agent in round t):

$$\delta D_{nc} + (1 - P_{(n-1)\alpha})D_{nc} + P_{(n-1)\alpha}D_c + P_{n-1}[b - (1 - P_{(n-1)\alpha})\triangle]$$

where δD_{nc} is the present value (in round $n-1$) of environmental damages in round n, $(1-P_{(n-1)\alpha})D_{nc}+P_{(n-1)\alpha}D_c$ is the agency's expected cost in round $n-1$ if it does not inspect the private agent and $b-(1-P_{(n-1)\alpha})\triangle$ is the change in the agency's expected cost in round $n-1$ if it inspects the private agent.

Because $b > \triangle$ implies $b > (1 - P_{(n-1)\alpha})\triangle$, a rational agency optimally chooses $P_{n-1} = 0$ and will thus not inspect the private agent in round $n-1$ either. But then the private agent optimally does not comply in round $n-1$, and a rational agency will not inspect in round $n-2$, and so forth.

Thus:

Lemma 3.1 *If during round t, with $t < n$, a rational agency does not inspect the private agent, then the private agent will never comply in the subsequent rounds and the agency will never inspect it.*

To simplify notation, we shall no longer specify the players' optimal actions and beliefs once the enforcement agency has not inspected the private agent.

Suppose now however that the private agent has been inspected in the first t rounds. It will then have to update its beliefs according to Bayes' rule.

The private agent's updated belief ν_t that the agency is type I if the agency has inspected the private agent in the first $t-1$ rounds is:

$$\frac{1.\nu_{t-1}}{1.\nu_{t-1} + p_{t-1}(1-\nu_{t-1})}$$

where p_{t-1} is the equilibrium probability that a type II agency inspects the private agent in the round $t-1$, *conditionally* on having inspected the private agent in all previous rounds.

The private agent's updated belief ν_{t-1} that the agency is type I if it has inspected the private agent in the first $t-2$ rounds is:

$$\frac{1.\nu_{t-2}}{1.\nu_{t-2} + p_{t-2}(1-\nu_{t-2})}$$

where p_{t-2} is the equilibrium probability that a type II agency inspects the private agent in the round $t-2$, *conditionally* on having inspected the private agent in all previous rounds.

Through recursion, we thus obtain:

$$\nu_t = \frac{1.q_0}{1.q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)} \tag{1}$$

where p_i is the equilibrium probability that a type II agency inspects the private agent in the round i, *conditionally* on having inspected the private agent in all previous rounds. Obviously, the private agent's updated belief that the agency is type II if it has inspected the private agent in the first $t-1$ rounds is $\frac{(\Pi_{i=1}^{t-1} p_i)(1-q_0)}{q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)}$.

The private agent's belief that it will be inspected in round $t > 1$, *conditionally* on having been inspected in all previous rounds, is then:

$$\frac{q_0}{q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)} + \frac{(\Pi_{i=1}^{t-1} p_i)(1-q_0)}{q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)} p_t = \frac{q_0 + (\Pi_{i=1}^{t} p_i)(1-q_0)}{q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)}$$

Indeed, the private agent will either be inspected because the agency is a type I agency (and thus inspects the private agent with certainty) or because it is a type II agency (and thus inspects the private agent with probability p_t).

The results we have obtained until now allow us to identify the private agent's expected costs at the beginning of each round of the game.

Indeed, if the private agent complies in round t, then its expected costs in that round are its compliance costs (α). If the private agent does not comply in round $t > 1$, then its expected costs: $\frac{q_0+(\Pi_{i=1}^{t}p_i)(1-q_0)}{q_0+(\Pi_{i=1}^{t-1}p_i)(1-q_0)}(\theta\alpha+\Psi)$.

Let $p_{t\alpha}$ be the probability that the private agent complies in round t, conditionally on having been inspected in all previous rounds. *If it has been inspected in all previous rounds*, then the private agent's expected cost in round $t > 1$ are:

$$\begin{aligned}(1-p_{t\alpha})\frac{q_0+(\Pi_{i=1}^{t}p_i)(1-q_0)}{q_0+(\Pi_{i=1}^{t-1}p_i)(1-q_0)}(\theta\alpha+\Psi)+p_{t\alpha}\alpha &= \\ \frac{q_0+(\Pi_{i=1}^{t}p_i)(1-q_0)}{q_0+(\Pi_{i=1}^{t-1}p_i)(1-q_0)}(\theta\alpha+\Psi)+p_{t\alpha}[\alpha-\frac{q_0+(\Pi_{i=1}^{t}p_i)(1-q_0)}{q_0+(\Pi_{i=1}^{t-1}p_i)(1-q_0)}(\theta\alpha+\Psi)] \quad (2)\end{aligned}$$

In round 1, expected costs for the private agent are:

$$(q_0+(1-q_0)p_1)(\theta\alpha+\Psi)+p_{1\alpha}[\alpha-(q_0+(1-q_0)p_1)(\theta\alpha+\Psi)] \quad (3)$$

If $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$ (this is possible because $\Psi > (1-\theta)\alpha$), then:

$$\begin{aligned}(q_0+(1-q_0)p_1)(\theta\alpha+\Psi) &= ((1-p_1)q_0+p_1)(\theta\alpha+\Psi) \\ &> ((1-p_1)\frac{\alpha}{\theta\alpha+\Psi}+p_1)(\theta\alpha+\Psi) \\ &= \alpha+p_1(\Psi-(1-\theta)\alpha) \\ &\geq \alpha\end{aligned}$$

This implies immediately:

Lemma 3.2 *If $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$, then the private agent always complies in the first round.*

If a type II agency has inspected the private agent in all rounds preceding round t, then the sum of its expected costs for all subsequent rounds, discounted at time t, can be found as follows.

At the start of round t, the agency expects the following costs:

- In round t:

$$\begin{aligned}(1-p_{(t)\alpha})D_{nc}+p_{(t)\alpha}D_c+p_t(b-(1-p_{(t)\alpha})\triangle) &= \\ D_{nc}-p_{(t)\alpha}(D_{nc}-D_c)+p_t(b-(1-p_{(t)\alpha})\triangle)\end{aligned}$$

- In round $t+i$ (with $i > 0$):

$$(1 - (\Pi_{j=t}^{t+i-1} p_j) p_{(t+i)\alpha}) D_{nc} + (\Pi_{j=t}^{t+i-1} p_j) p_{(t+i)\alpha} D_c + (\Pi_{j=t}^{t+i} p_j)(b - (1 - p_{(t+i)\alpha})\triangle) =$$
$$D_{nc} - (\Pi_{j=t}^{t+i-1} p_j) p_{(t+i)\alpha}(D_{nc} - D_c) + (\Pi_{j=t}^{t+i} p_j)(b - (1 - p_{(t+i)\alpha})\triangle)$$

- In round n (remember that in round n, it is a dominated strategy to inspect the private agent):

$$(1 - (\Pi_{j=t}^{n-1} p_j) p_{n\alpha}) D_{nc} + (\Pi_{j=t}^{n-1} p_j) p_{n\alpha} D_c =$$
$$D_{nc} - (\Pi_{j=t}^{n-1} p_j) p_{n\alpha}(D_{nc} - D_c)$$

Taking the sum of the discounted values of these expressions, we obtain:

$$\frac{1-\delta^{n-t+1}}{1-\delta} D_{nc} + \sum_{i=t}^{n-1} [(\Pi_{j=t}^{i} p_j)\delta^{i-t}(b - (1 - p_{i\alpha})\triangle)]$$
$$-p_{t\alpha}(D_{nc} - D_c) - \sum_{i=t+1}^{n} [(\Pi_{j=t}^{i-1} p_j)\delta^{i-t} p_{i\alpha}(D_{nc} - D_c)]$$

Relabeling the indices allows to obtain Equation 4:

$$\frac{1-\delta^{n-t+1}}{1-\delta} D_{nc} - p_{t\alpha}(D_{nc} - D_c)$$
$$+ \sum_{i=t+1}^{n} (\Pi_{j=t}^{i-1} p_j)\delta^{i-1-t}[b - \delta p_{i\alpha}(D_{nc} - D_c) - (1 - p_{(i-1)\alpha})\triangle] \tag{4}$$

From Equations 2 and 4, it is clear that the possible equilibria will be determined by the relative values of q_0 and $\frac{\alpha}{\theta\alpha+\Psi}$ on the one hand and b, $\triangle$ and $\delta(D_{nc} - D_c)$ on the other hand.

4 The default result

Proposition 4.1 *If $b > \triangle$, then the following actions are the equilibrium path of a PBE: the agency never inspects the private agent, the private agent complies in the first round iff $q_0 > \frac{\alpha}{\Psi+\theta\alpha}$ and does not comply in any subsequent round.*

Proof

If we substitute $p_{i\alpha} = 0$ and $p_{(i-1)\alpha} = 0$ (with $i > 2$) in Equation 4, then the agency's expected costs at the beginning of the inspection game are:

$$\frac{1-\delta^{n-t+1}}{1-\delta} D_{nc} - p_{1\alpha}(D_{nc} - D_c) + p_1(b - (1 - p_{1\alpha})\triangle) + \sum_{i=3}^{n} \delta^i (\Pi_{j=1}^{i-1} p_j)(b - \triangle)$$

If $b > \triangle$, then the agency optimally chooses $p_j = 0 \forall j$.

If the agency chooses $p_j = 0 \forall j$, then the private agent optimally never complies from the second round on.

In round 1, expected costs for the private agent are:

$$q_0(\Psi + \theta\alpha) + p_{1\alpha}[\alpha - q_0(\Psi + \theta\alpha)]$$

and the private agent optimally complies in the first round iff $q_0 > \frac{\alpha}{\Psi + \theta\alpha}$. □
QED □

Comments

Remember that the game we consider here is not a repeated game : the private agent's actions can only be observed if the agency inspects the private agent. The agency can thus only condition its play on the private agent's behavior if it has always inspected the private agent. However, this corresponds to a well-known result in the theory of repeated interactions: except for the first round *any equilibrium in the stage game also constitutes an equilibrium of the repeated game.* Note that *the private agent's behavior in the first round does depend on the fine!*

This implies immediately that whatever the value of the parameters, this equilibrium exists besides the equilibria we shall identify in the rest of this text. Moreover, for some parameter values, we have been unable to find another equilibrium. This result thus counts as the default result for our analysis.

5 Equilibrium if $b > \triangle + \delta(D_{nc} - D_c)$

Proposition 5.1 *If $b > \triangle + \delta(D_{nc} - D_c)$, then the* only *actions that survive iterated elimination of strictly dominated strategies are: the agency never inspects the private agent, the private agent complies in the first round iff $q_0 > \frac{\alpha}{\Psi + \theta\alpha}$ and does not comply in any subsequent round.*

Proof

Because $b > \triangle + \delta(D_{nc} - D_c)$:

$$\begin{aligned} b - \delta p_{i\alpha}(D_{nc} - D_c) - (1 - p_{(i-1)\alpha})\triangle &> \delta(1 - p_{i\alpha})(D_{nc} - D_c) + p_{(i-1)\alpha}\triangle \\ &\geq 0 \end{aligned}$$

This implies that the agency's cost function is minimized when $\forall i : \Pi_{j=t}^{i-1} p_j = 0$.

As this is true for any $t \geq 1$ and for any strategy played by the private agent, the agency optimally chooses $p_1 = 0$, but then the private agent knows from the first round on that the agency is rational and it will never comply from round 2 on.

The private agent's expected cost in round $t = 1$ is:

$$q_0(\theta\alpha + \Psi) + p_{1\alpha}[\alpha - q_0\ (\theta\alpha + \Psi)]$$

This means that the private agent will comply in the round 1 if $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$. □ QED □

Comments

In order to understand this result, remember that if the agency inspects the private agent in round t, then it obtains an immediate gain $\triangle$ if the private agent does not comply in that round t. Moreover, inspecting the private agent may convince the private agent that the agency will also inspect it in the next round. If the private agent believes this, then it will comply, and the present value (discounted to round t) of this benefit is $\delta(D_{nc} - D_c)$.

Thus, if $b > \triangle + \delta(D_{nc} - D_c)$, then the cost of inspecting a private agent is too high compared to the maximal possible benefit of this inspection and repeated interactions will not change anything.

Finally, *the fine for noncompliance* and the private compliance cost *affect the agency's cost in equilibrium* indirectly, because they determine the private agent's behavior in the first round. Indeed, if $q_0 > \frac{\alpha}{\Psi+\theta\alpha}$, then the agency's expected costs are $D_c + \delta\frac{1-\delta^{n-t+1}}{1-\delta}D_{nc}$; if $q_0 < \frac{\alpha}{\Psi+\theta\alpha}$, then the agency's expected costs are $\frac{1-\delta^{n-t+1}}{1-\delta}D_{nc}$.

6 Equilibrium if $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$ and $\delta(D_{nc}-D_c) > b > \triangle$

Proposition 6.1 *If $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$ and $\delta(D_{nc} - D_c) > b > \triangle$, then the following actions are the equilibrium path of a PBE: the agency inspects the private agent in all but the last round and the private agent complies in all rounds.*

Proof

If $p_t = 1$, for all $t < n$, then $\nu_t = q_0$ for all t.

If the agency has inspected the private agent in all previous rounds, then the private agent's expected cost in round $t = n$ is:

$$q_0(\theta\alpha + \Psi) + p_{t\alpha}[\alpha - q_0\ (\theta\alpha + \Psi)]$$

This means that the private agent will comply in round n if and only if $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$.

If the agency has inspected the private agent in all previous rounds, then the private agent's expected cost in round $t < n$ is:

$$\Psi + p_{t\alpha}[\alpha -\ (\theta\alpha + \Psi)]$$

This means that the private agent will comply in round $t < n$.

If, conditionally on always having been inspected, the private agent complies in all future rounds and if the agency has always inspected the private agent in previous rounds, then the sum of the agency's expected costs for all future rounds in round t are:

$$\frac{1-\delta^{n-t+1}}{1-\delta}D_{nc}-(D_{nc}-D_c)+\sum_{i=t+1}^{n}[(\Pi_{j=t}^{i-1}p_j)\delta^{i-1-t}[b-\delta(D_{nc}-D_c)]$$

If $\delta(D_{nc}-D_c)>b$, then the agency must choose $\Pi_{j=t}^{i-1}p_j=1$ for all $j<n$. Following a recursive argument, the agency optimally chooses thus $p_j=1$ if $j<n$.

Finally, as a type II agency *never* inspects the private agent in the last round, this is indeed a PBE. □ QED □

Comments

This means that if the a-priori belief that the agency always inspects the private agent is high enough ($q_0>\frac{\alpha}{\theta\alpha+\Psi}$) and the cost of inspecting the private agent is low enough compared to the discounted external benefit of spontaneous compliance ($\delta(D_{nc}-D_c)>b$), then the agency can obtain *perfect compliance* through repeated interactions. Note also that this equilibrium is only possible if $\delta(D_{nc}-D_c)>\triangle$, thus if the discounted benefit of spontaneous compliance is higher than the benefit of an immediate inspection of a noncompliant private agent.

Note that the agency's expected discounted costs at the beginning of the game are $\frac{1-\delta^n}{1-\delta}[b+D_c]$. Expected discounted costs converge thus to $\frac{1}{1-\delta}[b+D_c]$ when $n\to\infty$. Again, *the agency's cost function is affected indirectly by the fine* and by private compliance costs: they determine whether this equilibrium is possible.

7 Equilibria in pure strategies if $\frac{\alpha}{\theta\alpha+\Psi}>q_0$

Proposition 7.1 *If $\frac{\alpha}{\theta\alpha+\Psi}>q_0$, then the following actions are the equilibrium path of the only PBE in* pure *strategies: the agency never inspects the private agent; the private agent never complies.*

Part 1

First we show that this is indeed the equilibrium path of a PBE.

If it has never been inspected, then the private agent knows that the agency is type II and it optimally never complies from the second round on.

In round 1, expected costs for the private agent are:

$$q_0(\theta\alpha+\Psi)+p_{1\alpha}[\alpha-q_0(\theta\alpha+\Psi)]$$

and the private agent optimally never complies in the first round iff $\frac{\alpha}{\theta\alpha+\Psi} > q_0$.

If the private agent never complies, then the agency's expected costs from round t on (if it has inspected until round t) reduce to:

$$\frac{1-\delta^{n-t+1}}{1-\delta}D_{nc} + \sum_{i=t}^{n}(\Pi_{j=t+1}^{i-1}p_j)\delta^{i-1-t}(b-\triangle)$$

The agency thus optimally chooses $p_t = 0$ for all t. □ QED □

Part 2

Now we show that this is indeed the only possible PBE in pure strategies.

First note that the agency never inspects the private agent in the last round.

Now suppose that the agency has inspected the private agent with probability 1 in the first $n-1$ rounds. From the Proof of Proposition 6.1, we know that if $\frac{\alpha}{\theta\alpha+\Psi} > q_0$, then the private agent will not comply in round n. But then the agency optimally never inspects the private agent in round $n-1$!

Next suppose that the agency has inspected the private agent with probability 1 in the first $n-2$ rounds and does not inspect the private agent in the last two rounds. This implies that in the last round, the private agent knows that the agency is type II and it will not comply. Following the same logic as in the Proof of Proposition 6.1, if the private agent has always been inspected until round $n-2$ but $\frac{\alpha}{\theta\alpha+\Psi} > q_0$, then the private agent will not comply in round $n-1$ either. But if the private agent does not comply in any of the two last rounds, then the agency optimally never inspects the private agent in round $n-2$!

This argument can be repeated for any number of rounds.

As we know already that the agency will never start inspecting again once it has skipped an inspection, this shows that no PBE is possible where the private agent is ever inspected with certainty. □ QED □

Comments

This shows that if the a-priori belief that the agency always inspects the private agent is low enough, then it is impossible to obtain compliance if the agency and the private agent are only allowed to play pure strategies. However, we now show that if the cost of inspecting the private agent is low enough, then appropriate mixing can in the limit lead to the same expected costs for the agency as when $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$.

8 Equilibria in behavioral strategies if $\frac{\alpha}{\theta\alpha+\Psi} > q_0 > (\frac{\alpha}{\theta\alpha+\Psi})^n$ and $\delta(D_{nc} - D_c) > b > \triangle$

Let $\tau = n - \frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}}$.

Before proceeding further, note that $1 > \frac{\alpha}{\theta\alpha+\Psi} > q_0$ implies $0 > \ln\frac{\alpha}{\theta\alpha+\Psi} > \ln q_0$ and thus, $\frac{\ln q_0}{\ln\frac{\alpha}{\theta\alpha+\Psi}} > 1$, which in turn implies $n-1 > \tau$.

In general, τ is not a natural number. Let τ_1 be the smallest natural number such that $\tau_1 \geq \tau$.

Proposition 8.1 *If $\delta(D_{nc}-D_c) > b > \triangle$ and $\frac{\alpha}{\Psi+\theta\alpha} > q_0 > (\frac{\alpha}{\Psi+\theta\alpha})^n$, then the following actions are the equilibrium path of a PBE:*

- *If $t < \tau_1$, then the agency always inspects the private agent in round t; if $t = \tau_1$, then the agency inspects the private agent with probability $\frac{\alpha((\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1})}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}}$; if $n > t > \tau_1$ and if it has always inspected the private agent until then, then the agency inspects the private agent with probability $\frac{\alpha((\theta\alpha+\Psi)^{n-t}-\alpha^{n-t})}{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}}$ in round t; the agency never inspects the private agent in the last round or if it has not inspected the private agent in one or more previous rounds.*
- *The private agent complies as long as $t < \tau_1$; the private agent complies with probability $\frac{b}{\delta(D_{nc}-D_c)}$ if $t = \tau_1$; the private agent complies with probability $p_{t\alpha} = \frac{b-(1-p_{(t-1)\alpha})\triangle}{\delta(D_{nc}-D_c)}$ if $\tau_1 < t \leq n$. The private agent stops complying if it has not been inspected during at least one previous round.*

Proof

Part 1 - The private agent optimally reacts to the agency's behavior

It can be easily be verified that $\Psi > (1-\theta)\alpha$ implies that $1 > \frac{\alpha((\theta\alpha+\Psi)^{n-t}-\alpha^{n-t})}{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}} > 0$.

Next note that $\frac{\alpha((\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1})}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}} < 1$ as well. Indeed, $\tau_1+1 > \tau$ implies that $(\theta\alpha+\Psi)^{n-\tau+1}-\alpha^{n-\tau+1} > (\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1}$ (to see this, $\frac{d((\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau})}{d\tau} = -(\ln(\theta\alpha+\Psi))\ (\theta\alpha+\Psi)^{n-\tau} + (\ln\alpha)\ \alpha^{n-\tau} < 0$), and thus $\frac{\alpha((\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1})}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}} < \frac{\alpha((\theta\alpha+\Psi)^{n-\tau+1}-\alpha^{n-\tau+1})}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}} < 1$.

If the agency plays its equilibrium strategy, the private agent's expected cost in round t such that $\tau_1 > t \geq 1$ is (taking into account that $\Pi_{i=1}^{t}p_i = 1 = \Pi_{i=1}^{t-1}p_i$) :

$$\theta\alpha+\Psi+p_{t\alpha}[\alpha-(\theta\alpha+\Psi)]$$

Because $\Psi > (1-\theta)\alpha$, the private agent always complies as long as $\tau_1 > t$.

Now note that if $t \geq \tau_1$:

$$\Pi_{i=1}^{t}p_i =$$
$$\frac{\alpha((\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1})}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}}\frac{\alpha((\theta\alpha+\Psi)^{n-\tau_1-1}-\alpha^{n-\tau_1-1})}{(\theta\alpha+\Psi)^{n-\tau_1}-\alpha^{n-\tau_1}}\cdots\frac{\alpha((\theta\alpha+\Psi)^{n-t}-\alpha^{n-t})}{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}} =$$

$$\alpha^{t-\tau}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{(\theta\alpha+\Psi)^{n-\tau}-\alpha^{n-\tau}} =$$
$$\alpha^{t-n}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{(\frac{\Psi+\alpha}{\alpha})^{n-\tau}-1}$$

$\tau = n - \frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}}$ implies:

$$\begin{aligned}
\Pi_{i=1}^t p_i &= \\
\alpha^{t-n}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{(\frac{\Psi+\alpha}{\alpha})^{\frac{\ln q_0}{\ln\frac{\alpha}{\Psi+\alpha}}}-1} &= \\
\alpha^{t-n}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\exp^{(\ln\frac{\Psi+\alpha}{\alpha})\frac{\ln q_0}{\ln\frac{\alpha}{\Psi+\alpha}}}-1} &= \\
\alpha^{t-n}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\exp^{-\ln q_0}-1} &= \\
\frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\alpha^{n-t}}
\end{aligned}$$

If it has been inspected in all previous rounds, the private agent's expected cost in round $t \in [\tau_1, n]$ is thus (substitute $\Pi_{i=1}^t p_i = \frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\alpha^{n-t}}$ in 2):

$$\begin{aligned}
&(\frac{q_0+\frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\alpha^{n-t}}(1-q_0)}{q_0+\frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}}{\alpha^{n-t+1}}(1-q_0)})(\theta\alpha+\Psi)+ \\
p_{t\alpha}[\alpha - &\frac{q_0+\frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t}-\alpha^{n-t}}{\alpha^{n-t}}(1-q_0)}{q_0+\frac{q_0}{1-q_0}\frac{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}}{\alpha^{n-t+1}}(1-q_0)}(\theta\alpha+\Psi)] = \\
\frac{\frac{(\theta\alpha+\Psi)^{n-t}}{\alpha^{n-t}}}{\frac{(\theta\alpha+\Psi)^{n-t+1}}{\alpha^{n-t+1}}}&(\theta\alpha+\Psi)+p_{t\alpha}[\alpha-\frac{\frac{(\theta\alpha+\Psi)^{n-t}}{\alpha^{n-t}}}{\frac{(\theta\alpha+\Psi)^{n-t+1}}{\alpha^{n-t+1}}}(\theta\alpha+\Psi)] = \\
&\alpha+p_{t\alpha}[\alpha-\alpha]
\end{aligned}$$

and the private agent is indeed indifferent with respect to the choice of $p_{t\alpha}$.

Part 2 - The agency optimally reacts to the private agent's behavior

First note that $\frac{b-(1-p_{(t-1)\alpha})\triangle}{\delta(D_{nc}-D_c)}$ is a probability if $p_{(i-1)\alpha}$ is a probability. Indeed $b > \triangle$ and $1 > p_{(i-1)\alpha} > 0$ imply $b > (1-p_{(i-1)\alpha})\triangle$. Moreover, $\delta(D_{nc}-D_c) > b$ implies that $1 > \frac{b-(1-p_{(t-1)\alpha})\triangle}{\delta(D_{nc}-D_c)}$ and that $p_{\tau_1} < 1$.

Now note that the private agent's equilibrium strategy implies that $b - p_{i\alpha}\delta(D_{nc}-D_c)-(1-p_{(i-1)\alpha})\triangle = 0$ for all $t \geq \tau_1$. This implies that the agency is indifferent with respect to its strategy choice for all $t \geq \tau_1$.

On the other hand, if $t < \tau$, the private agent's equilibrium strategy implies that the discounted sum of the agency's expected costs for all subsequent rounds in round t are:

$$D_c + \frac{\delta - \delta^{n-t+1}}{1-\delta} D_{nc} + \sum_{i=t+1}^{\tau} (\Pi_{j=t}^{i-1} p_j)\delta^{i-1-t}(b - \delta(D_{nc} - D_c))$$

If $\delta(D_{nc} - D_c) > b$, $p_j = 1$ is clearly the optimal choice for $t < \tau_1$.
□ QED □

Comments

Thus, if $\delta(D_{nc} - D_c) > b$ and $\frac{\alpha}{\theta\alpha+\Psi} > q_0 > (\frac{\alpha}{\theta\alpha+\Psi})^n$, then the agency's expected discounted costs at the beginning of the inspection game are (substitute the equilibrium probabilities directly in Equation 4):

$$\begin{aligned} & \frac{1-\delta^{n-t+1}}{1-\delta} D_{nc} - (D_{nc} - D_c) \\ & + \sum_{i=t+1}^{\tau_1 - 1} \delta^{i-1-t}[b - \delta(D_{nc} - D_c)] &= \\ -(D_{nc} - D_c) + \frac{1-\delta^{n+1}}{1-\delta} D_{nc} + & \frac{1-\delta^{\tau_1 - 1}}{1-\delta}[b - \delta(D_{nc} - D_c)] &= \\ \frac{-(D_{nc} - D_c) + (1-\delta^{n+1})D_{nc} + (1-\delta^{\tau_1-1})b + \delta^{\tau_1}(D_{nc} - D_c)}{1-\delta} & &= \\ \frac{D_c - \delta^{n+1}D_{nc} + (1-\delta^{\tau_1-1})b + \delta^{\tau_1}(D_{nc} - D_c)}{1-\delta} & & \qquad (5) \end{aligned}$$

We shall from now on note this expression as: $EC_{a,n}$.

Before proceeding further, it can be useful to understand why the agency has to switch to behavioral strategies.

Basically, what happens in repeated interactions, is that *the agency creates an intertemporal externality*: by inspecting a private agent, it tries to convince the private agent that it will continue to do so in the future. If the agency succeeds in doing this, then the private agent will comply in the future. Thus, by inspecting the private agent now, the agency creates future benefits.

In the beginning of the game, the discounted value of future benefits is high compared to the cost of inspecting a private agent with certainty during one more round. In the beginning of the game, the private agent will thus not be able to distinguish a type II agency from a type I agency. However, when the end of the game comes near, the discounted value of external benefits becomes smaller. In other words, *near the end, the advantage of mimicking the behavior of a type I agency becomes smaller. Because the private agent knows this, it will change its belief that it will be inspected with certainty in the future, even if it has always been inspected in the past.* Thus, the equilibrium probability of inspection can be interpreted here as the private agent's belief that the agency is type I (this is a standard interpretation of an equilibrium in mixed strategies, see for instance Osborne and Rubinstein (1994: 43-44).

Formally, the private agent's belief that the agency is a type I agency changes as follows with time:

$$\begin{aligned}\nu_{t+1} - \nu_t &= \\ \frac{q_0}{q_0 + (\Pi_{i=1}^{t} p_i)(1-q_0)} - \frac{q_0}{q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)} &= \\ \frac{q_0(1-q_0)(\Pi_{i=1}^{t-1} p_i)(1-p_t)}{[q_0 + (\Pi_{i=1}^{t} p_i)(1-q_0)][q_0 + (\Pi_{i=1}^{t-1} p_i)(1-q_0)]} &> 0\end{aligned}$$

Thus, the longer the private agent is inspected, the less it believes that the agency is type II, which corresponds to the interpretation we have given above.

Let us now turn to the comparative statics of the problem, keeping in mind that $D_{nc}-D_c > \delta(D_{nc}-D_c) > b$, $\ln q_0 < 0$, $\ln \frac{\alpha}{\theta\alpha+\Psi} < 0$, $\ln \delta < 0$ and $\frac{\alpha}{\theta\alpha+\Psi} > q_0$ (also note that $\frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}} > 0$ implies $\delta^{-\frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}}} > 1 \Rightarrow \delta^{-\frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}}}[(D_{nc}-D_c)-b] > [(D_{nc}-D_c)-b]$):

- We first verify how the number of interactions affects $EC_{a,n}$.

  First note that the definition of τ implies that $\tau_1(n+1) = \tau_1(n)+1$. Thus, if one round is added to the game, then the mixing also starts one round later.

  The difference in expected costs between a game with n rounds and an otherwise identical game with $n+1$ rounds is thus:

$$\begin{aligned}&\frac{D_c - \delta^{n+1}D_{nc} + (1-\delta^{\tau_1-1})b + \delta^{\tau_1}(D_{nc}-D_c)}{1-\delta} \\ &-\frac{D_c - \delta^{n+2}D_{nc} + (1-\delta^{\tau_1})b + \delta^{\tau_1+1}(D_{nc}-D_c)}{1-\delta} = \\ &\frac{\delta^{n+1}(\delta-1)D_{nc} + \delta^{\tau_1-1}(\delta-1)b + \delta^{\tau_1}(1-\delta)(D_{nc}-D_c)}{1-\delta} = \\ &-\delta^{n+1}D_{nc} - \delta^{\tau_1-1}b + \delta^{\tau_1}(D_{nc}-D_c) = \\ &\delta^n[-\delta D_{nc} - \delta^{\tau_1-n-1}b + \delta^{\tau_1-n}(D_{nc}-D_c)]\end{aligned}$$

  As $\tau_1 - n$ does not depend on n, the sign of this expression is not determined but does not depend on n.

  Thus, if $\delta^{\tau_1-n-1}[\delta(D_{nc}-D_c)-b] - \delta D_{nc} > 0$, then the agency's costs are minimized when $n \to \infty$. Expected discounted costs become then $\frac{1}{1-\delta}(D_c+b)$, the same expected costs as we obtained when $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$. Moreover, when $n \to \infty$, this equilibrium in behavioral strategies remains possible even when $q_0 \to 0$. Thus, however small the belief that the agency is type I, if there are enough interactions in the inspection game, then the private agent will comply in all but the last rounds. To put in still other words: *if the inspection game lasts long enough, then the agency can obtain permanent full compliance as long as $\delta(D_{nc} - D_c) > b$, however small the private agent's* a priori *beliefs are that the agency will always inspect.* This corresponds to conventional results in the literature on reputation effects (compare with Fudenberg and Tirole (1995: 372): "the size of the prior (...) required to deter entry shrinks geometrically as the number of period grows (...) even a small amount of incomplete information can have a very large effect in long games").

If $\delta^{\tau_1-n-1}[\delta(D_{nc} - D_c) - b] - \delta D_{nc} < 0$, then the agency's costs are minimized when $\tau_1 = 1$. n is then the largest natural number such that $n \leq 1 + \frac{\ln q_0}{\ln \frac{\alpha}{\theta\alpha+\Psi}}$. From Equation 5, the agency's expected discounted costs are then $\frac{D_c - \delta^{n+1}D_{nc} + b + (D_{nc} - D_c)}{1-\delta}$.

- We now verify how changes in the a-priori beliefs, private compliance costs and the fine affect $EC_{a,n}$.

  To see this, first rewrite Equation 5 as:

  $$\frac{D_c - \delta^{n+1}D_{nc} + b + \delta^{\tau_1-1}[\delta(D_{nc} - D_c) - b]}{1-\delta} \tag{6}$$

  Because $\delta(D_{nc} - D_c) > b$, we see that the agency's expected discounted costs increase when τ_1 increases (at least, if this change is not due to changes in n).

  This implies:

  - the smaller the private agent's *a priori* belief that the agency is type II, the lower expected discounted costs for the agency. Indeed, the smaller the private agent's *a priori* belief that the agency is type II, the later it will start believing that the agency will not inspect her:

    $$\frac{d\tau}{dq_0} = \frac{d(n - \frac{\ln q_0}{n \ln \frac{\alpha}{\theta\alpha+\Psi}})}{dq_0} = -\frac{1}{q_0 n \ln \frac{\alpha}{\theta\alpha+\Psi}} > 0$$

  - an increase in the compliance costs leads to an increase in expected discounted costs for the agency. Indeed, when compliance costs increase, the agency has to switch earlier to mixed strategies:

    $$\frac{d\tau}{d\alpha} = \frac{d(n - \frac{\ln q_0}{n \ln \frac{\alpha}{\theta\alpha+\Psi}})}{d\alpha} = \frac{\ln q_0}{n(\ln \frac{\alpha}{\theta\alpha+\Psi})^2} \frac{\Psi}{\alpha(\theta\alpha+\Psi)} < 0$$

Thus, *although the agency does not include private compliance costs in its objective function, they affect expected costs indirectly*, because they affect the round when the mixing starts.

 – an increase in the fine leads to a decrease in expected discounted costs for the agency. Indeed, the higher the fine, the longer the agency can wait before it starts playing mixed strategies:

$$\frac{d\tau}{d\Psi} = \frac{d(n - \frac{\ln q_0}{n \ln \frac{\alpha}{\theta\alpha+\Psi}})}{d\Psi} = \frac{\ln q_0}{n(\ln \frac{\alpha}{\theta\alpha+\Psi})^2} \frac{\Psi}{\alpha}(-\frac{\alpha}{\Psi^2}) = -\frac{\ln q_0}{n(\ln \frac{\alpha}{\theta\alpha+\Psi})^2} \frac{1}{\theta\alpha + \Psi} > 0$$

 Again, *the fine is not part of the agency's objective functions, but does affect its expected costs indirectly through the timing of the mixing.*

- Because $\frac{dEC_{a,n}}{db} = \frac{1-\delta^\tau}{1-\delta} > 0$, an increase in the cost of inspecting a private agent leads to an increase in expected costs for the agency.
- Finally, the agency's expected cost is the sum of positive discounted numbers and an increase in the discount rate leads to an increase in expected costs.

A few calculations also show that:

$$\begin{aligned} \frac{\alpha(\alpha^{n-t-1} - (\Psi + \theta\alpha)^{n-t-1})}{\alpha^{n-t} - (\Psi + \theta\alpha)^{n-t}} - \frac{\alpha(\alpha^{n-t} - (\Psi + \theta\alpha)^{n-t})}{\alpha^{n-t+1} - (\Psi + \theta\alpha)^{n-t+1}} &= \\ -\frac{\alpha^{n-t}(\Psi + \theta\alpha)^{n-t-1}(\Psi - (1-\theta)\alpha)^2}{[\alpha^{n-t} - (\Psi + \theta\alpha)^{n-t}][\alpha^{n-t+1} - (\Psi + \theta\alpha)^{n-t+1}]} &< 0 \end{aligned}$$

Thus, once the agency starts playing a mixed strategy, the conditional probability of inspection declines with time. This is consistent with the result that the private agent's belief that the agency is type I increases with time.

Finally:

Proposition 8.2 *The conditional probability of compliance declines with time*

Proof

We need to show:

$$p_{t\alpha} = \frac{b - (1 - p_{(t-1)\alpha})\triangle}{\delta(D_{nc} - D_c)} \quad < \quad p_{(t-1)\alpha}$$

This condition is equivalent with:

$$\frac{b - \triangle}{\delta(D_{nc} - D_c) - \triangle} \quad < \quad p_{(t-1)\alpha}$$

First note that $\delta(D_{nc}-D_c) > b$ implies immediately that $p_{\tau_1\alpha} = \frac{b}{\delta(D_{nc}-D_c)} > \frac{b-\triangle}{\delta(D_{nc}-D_c)-\triangle}$.

Suppose now that there exists an i such that:

$$p_{i\alpha} < \frac{b-\triangle}{\delta(D_{nc}-D_c)-\triangle} < p_{(i-1)\alpha}$$

This would imply:

$$\begin{aligned} b - \delta p_{i\alpha}(D_{nc}-D_c) - (1-p_{(i-1)\alpha})\triangle &> \\ b - \delta\frac{b-\triangle}{\delta(D_{nc}-D_c)-\triangle}(D_{nc}-D_c) - \triangle + \frac{b-\triangle}{\delta(D_{nc}-D_c)-\triangle}\triangle &= 0 \end{aligned}$$

The agency would then not be indifferent between inspecting and not inspecting, and this could never be part of an equilibrium in behavioral strategies. □ QED □

9 Equilibrium with commitment

As we have stated in Section 1, most inspection models assume that the enforcement agency can commit to announced inspection probabilities. This means that the agency inspects the private agents according to these announced probabilities, even if it is not optimal *ex post* to stick to it if the private agents think that the agency's threat is credible. For instance, if the announced inspection policy induces the private agents to comply, then it is clearly optimal not to inspect the private agents (at least in a one-shot interaction).

A typical argument in favor of this assumption of commitment would be that in a long-term relationship, the inspection agency wants to develop a reputation that it will inspect the private agents according to the announced inspection probabilities (see, for instance, Fudenberg and Tirole (1995: 368- 395)). We shall now verify this claim in the particular context we are studying.

Therefore, we use the framework we have developed in Section 2, but assume from now on that the enforcement agency can commit.

The timing of the game is:

- The enforcement agency announces the probability of inspecting the private agent.
- The private agent chooses whether or not it complies.
- The agency inspects according to its *announced* strategy. If the private agent is found in noncompliance, then he has to pay the fine.

Because the private agent believes that the agency will stick to the announced inspection probability (this is exactly the idea of commitment), he knows this probability with certainty before choosing whether or not he will comply and the relevant solution concept is a subgame-perfect Nash equilibrium (SPE). We must thus start by looking at the private agents' reaction to an *announced* inspection probability.

Let p^α be the probability that the private agent complies and let p_i be the announced probability of inspection. From the payoff-matrix (see Table 1), the private agent's expected cost is:

$$p^\alpha \alpha + (1 - p^\alpha) p_i(\theta\alpha + \Psi) = p_i(\theta\alpha + \Psi) + p^\alpha[\alpha - p_i(\theta\alpha + \Psi)]$$

Expected costs are linear in p^α. They are thus increasing in p^α iff $\alpha > p_i(\theta\alpha + \Psi)$. Thus, the private agent will comply if $p_i > \frac{\alpha}{\theta\alpha+\Psi}$, will not comply if $\frac{\alpha}{\theta\alpha+\Psi} > p_i$ and will be indifferent if $p_i = \frac{\alpha}{\theta\alpha+\Psi}$.

From now on, we shall simplify notation and assume that if the private agent is indifferent between complying and not complying, then he will comply.

We can thus consider two possible inspection policies:

- If $p_i \geq \frac{\alpha}{\theta\alpha+\Psi}$, then the private agent complies, and the agency minimizes expected costs by minimizing inspection costs, in other words by setting $p_i = \frac{\alpha}{\theta\alpha+\Psi}$. In this case, the agency's expected costs are (the first term are expected inspection costs, the second term are expected external damages): $\frac{\alpha}{\theta\alpha+\Psi} b + D_c$.
- If $\frac{\alpha}{\theta\alpha+\Psi} > p_i$, then the private agent does not comply and the agency's expected costs are: $D_{nc} + p_i b$. Because $b > 0$, the agency minimizes expected costs by setting $p_i = 0$. Its expected costs are then: D_{nc}.

From the analysis above, the agency will choose $p_i = \frac{\alpha}{\theta\alpha+\Psi}$ iff $\frac{\alpha}{\theta\alpha+\Psi} b + D_c < D_{nc}$.

We can thus conclude:

Proposition 9.1 *If $D_{nc} - D_c > \frac{\alpha}{\theta\alpha+\Psi} b$, the following strategies form a SPE: the agency announces $p_i = \frac{\alpha}{\theta\alpha+\Psi}$, the private agent complies and the enforcement agency inspects according to the announced probabilities.*

Note that in this equilibrium, the private agent will be inspected with a positive probability, even though he complies. The agency's actions are thus not optimal *ex post*.

On the other hand:

Proposition 9.2 *If $\frac{\alpha}{\theta\alpha+\Psi} b > D_{nc} - D_c$, the following strategies form a subgame-perfect equilibrium: the agency announces not to inspect the private agent, the private agent never complies and the agency never inspects.*

Thus, *if the agency can commit to announced inspection probabilities and if the fine is high enough, then it can obtain perfect compliance.* Again, keep in mind that these results are only valid if $\Psi \geq 1(1 - \theta)\alpha$.

From these two results, it is clear that it is difficult to justify our commitment model as the limit of a game with reputation effects.

Indeed, we have shown that the agency will either commit not to inspect the private agents, either to inspect them with a given probability. In our model with reputation effects, however, the agency tries to develop a reputation for undertaking certain *actions* with certainty, not for inspecting with a certain *probability* (except in Section 8). Now take a closer look at the behavioral equilibria treated in Section 8. As $\lim_{n\to\infty} \frac{\alpha((\theta\alpha+\Psi)^{n-t}-\alpha^{n-t})}{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}} = \frac{\alpha}{\theta\alpha+\Psi}$, in the limit, the equilibrium conditional probability of inspection in the reputation game converges to the equilibrium probability of inspection in the commitment game. One could thus be tempted to say that our model with reputation effects does provide a justification for the commitment hypothesis if one assumes an infinity of interactions. This is however not correct, for two reasons. First, if $n \to \infty$, then $\tau \to \infty$ as well, and the agency never mixes in the limit. Second, $\frac{\alpha((\theta\alpha+\Psi)^{n-t}-\alpha^{n-t})}{(\theta\alpha+\Psi)^{n-t+1}-\alpha^{n-t+1}}$ are *conditional* probabilities of inspection. Once the agency has not inspected the private agent during one round, it never inspects the private agent again in the future.

10 Conclusion

The analysis in this paper shows that an enforcement agency can obtain compliance in repeated interactions where it would not obtain this in a non-repeated interaction. Through reputation effects, this is possible, even with a finite number of interactions.

We can summarize the results we have obtained as follows:

- If $b > \triangle + \delta(D_{nc} - D_c)$, then it is impossible for the agency to obtain compliance with a finite number of interactions; the agency never inspects the private agent.
- If $q_0 > \frac{\alpha}{\theta\alpha+\Psi}$ and $\delta(D_{nc} - D_c) > b$, then there exists an equilibrium where the agency inspects the private agent in all but the last round and the private agent complies in all rounds.
- If $\frac{\alpha}{\theta\alpha+\Psi} > q_0$ and $\delta(D_{nc} - D_c) > b$, then no PBE in pure strategies is possible where the agency always inspects the private agent. However, if the agency starts mixing a few rounds before the final round after having inspected the private agent with certainty in all previous rounds, then an equilibrium is possible where the private agent first complies with certainty and starts mixing once the agency has started mixing herself. As long as the game lasts long enough, this equilibrium is always possible, however small the private agent's *a priori* belief that it will always be inspected.
- Contrary to what we would expect, our model with reputation effects does *not* converge to a model with perfect commitment when the number of interactions becomes very large.
- Through the constraints they impose on the private agent's equilibrium behavior, the fine for noncompliance and the private compliance costs

always affect the agency's cost function and behavior. This result suggests that Tsebelis's (1990,1993) claim that penalties do not matter does not necessarily carry over to a dynamic context.

We should however keep in mind that these results are only valid if $\Psi > (1-\theta)\alpha$. If there exist stringent upper limits on the fine that can be levied, then repeated interactions can still allow improvements compared to the static game if the agency also conditions its play on the private agent's past performance (see, for instance, Harrington (1988)); we shall not further explore this possibility here.

There are however a few other problems with the preceding analysis.

First, we assumed an exogenous number of interactions.

One possible way to justify this assumption is to assume that the management of the enforcement agency has a limited tenure, or that the number of interactions corresponds to time remaining before the next elections.

This then raises the problem how we should treat what lies ahead of the inspection game. The private agent should then also take into account how its behavior will affect the behavior of the next enforcement agency (or government), but without knowing the identity and objectives of the next opponent.

An alternative approach would then be to consider from the outset a game with an infinity of interactions. The discount rate would then incorporate the uncertainty with respect to the continuation of the game (for instance, because the cost of the abatement technology or estimated external damages change, or because there are preliminary elections).

Another problem is the definition of the time unit.

We have assumed that a type I agency inspects the private agent with certainty during the depreciation period of the abatement equipment. This is clearly a problematic assumption. We have no reason to assume that the budget the agency faces allows to inspect all private agents in that time period (or that the private agent cannot be inspected twice during that period). Moreover, nothing changes compared to the static game if the discount factor is low enough, in other words if the time period between two inspections is long enough.

A more realistic formulation of the problem would be to consider a setting where a type I agency inspects the private agent with a given probability, rather than with certainty, or where the depreciation period does not correspond to the time interval between two inspections by a type I agency. This however raises a new problem: how can the private agent *observe* this probability that will condition his future play? Fudenberg and Levine (1989) have obtained results in reputation models where the players are allowed to play mixed strategies, but as we have already explained in Section 1, their model is thoroughly different from ours. This could thus be a fruitful area for further research.

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Notes

[1]See for instance Polinsky and Shavell (2000: 49).

[2]Similar arguments have been used by Grieson and Singh (1990).

[3]The general structure of this game is very close to those developed by Holler (1993), Saha and Poole (2000) and Tsebelis (1990a,1990b,1993).