Criminal behavior: a real option approach
With an application to restricting illegal insider trading

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Abstract
Traditionally, criminal behavior is analyzed within an expected utility framework. This paper offers an alternative model to analyze criminal behavior based on real option models. It is shown that all criminal decisions can be analyzed as real options, in a sense that they confer the possibility but not the obligation to commit a crime in the future. The criminal option model is a richer model compared to conventional economic models of crime, because it takes into account four additional variables. The criminal option model is then applied to the enforcement of illegal insider trading. Based on the six value-drivers of criminal options, an active management strategy can be developed for both the criminal as for the legislator.

Keywords: insider trading, enforcement, criminal behavior, real options

JEL classification: K42

0.Introduction

Although it is a plausible hypothesis to assume that investors and companies care about the quality of the financial market in which they operate, the so-called law and finance literature, which was initiated by the seminal papers of La Porta, Lopez-de-Silanes, Shleifer and Vishny (1997, 1998), only recently investigated the relationship between a country's legal framework and its financial development. The law and finance literature offers strong empirical evidence on the importance of the legal environment (market integrity, investor protection) for the development of these markets and economic growth. La Porta e.a. (1997) show that a good legal environment expands the ability of companies to raise external finance through either debt or equity. The European Commission as well stresses the importance of an adequate legal framework in order for companies to raise capital[2]. Although the existence of legal rules is an important element for the development of financial markets, La Porta e.a. (1998) show that the enforcement of these rules is of equal importance. The European Commission considers the enforcement of insider trading prohibition and market manipulation of crucial

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[2] European Commission, Explanatory Memorandum, Proposal for a Directive of the European Parliament and of the Council on Insider Dealing and Market Manipulation, 30 May 2001, COM(2001) 281 final.

importance to ensure the integrity of European financial markets and to enhance investor confidence in those markets[3].

Although the prohibition of insider trading can be questionned on economic grounds[4], the question whether insider trading should be prohibited or not will not be adressed in this article. However, once the European legislator chooses to ban insider tradig, the question of the optimal deterrence of insider trading regulation arises. Engelen (1997) analyzed the enforcement of insider trading regulation within the classic expected utility framework of Becker (1968). This article extends this analysis by presenting a new, alternative model to analyze criminal behavior based on the recent real option literature.

Traditionally, criminal behavior is analyzed within an expected utility framework (Becker, 1968). The expected net-gain of a crime is the difference between the expected profits of a crime and the expected costs, being the product of the amount of punishment and its probability (Cooter and Ulen, 2000). The calculation of the expected net-gain of a crime is similar to the net present value (NPV) calculations in corporate finance. For instance, NPV is the traditional criterion to analyze the profitability of projects in capital budgeting. Only recently, many practitioners and academics recognized the limitations of the net present value model in capital budgeting or cost-benefit analysis. In particular, the NPV-analysis cannot capture flexibility to adapt an investment decision in response to uncertainty. A similar problem arises with respect to conventional economic analysis of criminal behavior (see infra). This gave rise to a large body of new literature, and a new class of models usually referred to as 'real options' models (Trigeorgis, 1996, 2000, Sick, 1995 and Amran and Kulatilaka, 1999).

This new theory is based on the simultaneous existence of three phenomena: uncertainty, irreversibility of investment and some freedom of choice on the timing of investment. In this way, a more dynamic framework to evaluate investment projects has emerged. Real option models have already been applied in a variety of business contexts, such as natural resource investments in oil and other commodities (Paddock, Siegel and Smith, 1988), land development (Quigg, 1994), flexible manufacturing (Kulatilaka, 1993), research and development

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[3] *Ibidem*.

[4] See e.g. Manne (1966), Carlton and Fischel (1983) and Engelen (2002). See also the review of literature of Scott (1998) and Bainbridge (2000).

(Cassimon and Engelen, 2000), leasing and others[5]. Besides the obvious business applications, real option theory can also be used in contexts such as global warming (Hendricks, 1991), environmental pollution compliance options (Edleson and Reinhardt, 1993), large-scale infrastructure projects of governments (Cassimon e.a., 2001), etc. In this chapter, real options models are applied to analyzing criminal behavior. We will show that all criminal decisions can be analyzed as real options, in a sense that they confer the possibility but not the obligation to commit a crime in the future.

The paper is organized as follows. After explaining the concept of real options in section one, while an overview of the general real option framework for criminal behavior is given in section two. This model will be applied to insider trading in section three. The next two sections analyze insider trading as a real option in more detail: section four examines this criminal option from the point of view of the criminal, while section five is the point of view of the legislator. How can insider trading be restricted based on the findings of an option model?

1.The concept of real options

Real option analysis teaches us that every investment project can be seen as exercising an option. In general, an option can be defined as the right, but not the obligation, to buy (call-option) or sell (put-option) the underlying asset at an agreed price (strike price or exercise price) during a specific period (as in the case of American options) or at a predetermined expiration date (as in the case of European options). Typical examples are stock options, index options, interest rate options, currency options or options on commodities.

The crucial insight of real option analysis is that the concept of options can also be applied in a real context, i.e. with respect to physical investment decisions, because these investment decisions also fit the general definition of what an option is. The investment project can be viewed as an option whereby the firm has the right to obtain all the underlying cash flows that are resulting from the investment project at a known price. This price is the investment cost of the project and analogous to the exercise price in financial options. When the firm decides to go along with the investment projects, it is in fact 'executing' this real option (Cassimon and Engelen, 1999). Table 1 explains shortly the different types of real options.

[5] See e.g. Trigeorgis (2000) for a recent overview.

Table 1.Different types of real options

Category	Description	Option type	Examples
Option to delay	The firm has some flexibility to delay the investment decision. Management has to determine the optimal timing of the investment. Should they invest now or wait until more information is available so that the investment decision can be made under less uncertainty?	Call	Titman (1985), Ingersoll and Ross (1992)
Growth option	An initial project is necessary to make future investment possible. This first stage project embodies options on later potential profitable projects. The first stage is a necessary, but not a sufficient condition for the next stage(s). These option characteristics must be incorporated in the evaluation of the first stage. The value of the real option on future investment opportunities should be added to the net present value of the initial project.	Call	Kester (1993), Trigeorgis (1988), Chung and Charoenwong (1991), Cassimon and Engelen (1999)
Scale option	If market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. If conditions are less favorable than expected, it can reduce the scale of operations.	Call	Trigeorgis and Masson (1987), Pindyck (1988)
Switch option	Situation in which a project creates the flexibility to switch between inputs (e.g. different energy sources –oil, gas or electricity) or outputs (e.g. PVC, PE or PP) due to a change in relative prices or market demand. This possibility should be valued as a call option with the additional cost as the exercise price.	Call	Kulatilaka (1988), Kulatilaka and Trigeorgis (1993)
Option to abandon	If market conditions decline severely, the firm can abandon current operations permanently and realize the salvage value of capital equipment and other assets in second-hand markets.	Put	Myers and Majd (1990)

There is a clear analogy between financial and real options. The six value drivers of a financial call-option on a stock can be translated in the six parameters in real option terminology[6]. Analogous to a financial option, the stock prices is transformed into the present value of all future free operating cash flows of the project (FOCFs). The exercise price at which the underlying asset can be acquired is in this case the investment expenditure of the project. Volatility is now measured as the standard deviation of the project return. Time to maturity is the duration of the real option (e.g. the investment can be postponed for two years). The risk-free interest rate is the same in both cases. Finally, the sixth value driver is the opportunity cost, this is the value lost during the duration of the option. For instance, by

[6] See also table 2.

postponing the investment decision, competitors can enter the market causing a decline of market share. The impact of the different parameters of real options is analogous to financial options. For instance, the present value of all future free operating cash flows (FOCFs) and volatility have a positive impact on the option value, while the investment expenditure has a negative impact (Hull, 2000).

The above analogy between financial and real options shows that the valuation models for real options build on the models for financial options, such as the model developed by Black and Scholes (1973) and Merton (1973), commonly referred to as the Black-Scholes model[7]. The value of real options according to this model can be calculated as:

$$C = V\, e^{-\delta(T-t)}\, N(d_1) - I\, e^{-r_c(T-t)}\, N(d_2) \text{, where} \quad [1]$$

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r_c - \delta + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad [2]$$

$$d_2 = \frac{\ln\left(\frac{V}{I}\right) + \left(r_c - \delta - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \text{, with:} \quad [3]$$

r_c = continuous risk-free interest rate,

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[7] We opt for using the Black-Scholes model to illustrate the real and criminal option approach because it is the most commonly used model in literature. It has a closed-form solution, which renders it computational simpler, and as such, it is easier to conduct sensitivity analysis (the 'Greeks'– see infra) using partial derivatives (Wilmott, 1998). The Black-Scholes analysis is based on the following assumptions (Hull, 2000, Trigeorgis, 1996 and Wolfgang and Baschnagel, 1999): (1) frictionless markets, meaning no transaction costs or taxes, no restrictions on short sales, and the underlying is arbitrarily divisible; (2) continuous trading is possible; (3) the risk-free (short-term) interest rate is constant over the life of the option; (4) the market is arbitrage-free; (5) the time process of the underlying asset price is stochastic and exhibits a geometric Brownian motion, as can be expressed by the following equation: $dS = \mu S\, dt + \sigma S\, dz$, according to which a price change dS in a small time interval dt consists of two components; a deterministic component (μ), also called the drift, which measures the average growth rate of the asset price and a random or stochastic component (σ), also called the volatility, which measures the strength of the statistical price fluctuations. This process assumes asset prices to be log-normally distributed and returns to be normally distributed. In reality stock returns are not perfectly normally distributed, because the geometric Brownian motion model predicts that large price movements are much less likely than they actually occur (leptokurtic or fat tails) (Jackwerth and Rubinstein, 1995). Therefore the Black-Scholes valuation may be slightly mispriced. However, it can still be used as a first approximation of the option value (Hull, 2000). If a more precise valuation is necessary, complex numerical procedures must be used instead of close-end models, such as Monte Carlo simulation (see Boyle, 1977) or lattice approaches (see Trigeorgis, 1991). For instance, if the asset price exhibits jumps, then these sudden price movements are too large to be from a normally distributed returns model, and an appropriate model must be used. A jump-model assuming a Poisson distribution must be used in such a case. See e.g. Pennings and Lint (1997). On models beyond geometric Brownian motion, see also section 5.3 in Wolfgang and Baschnagel (1999). Because we are less concerned about the exact valuation aspects of criminal options, but more focused on applying option-thinking compared to conventional economic analysis of crime, and because there is no specific reason to assume that the returns in criminal option models are not normally distributed, we will illustrate the criminal option approach by using the Black-Scholes model.

T-t = time to expiration,

N(d) = cumulative normal probability density function,

σ = standard deviation of the project return,

δ = opportunity cost,

V = present value of the future FOCFs, and

I = capital expenditure.

To illustrate the functioning of real option analysis, suppose a company considers the implementation of the following project. The project has an economic life span of 15 years, it yields yearly free operating cash flows of 120 million EUR, the initial investment expenditure amounts 800 million EUR, the risk-free interest rate and the discount rate of the project amount 10% and the uncertainty of the market demand for these products, as measured by the standard deviation, is estimated to be 10%. Suppose, management want to know whether to invest in this project immediately or to postpone the project with one year (while more information concerning the profitability of the project becomes available). This is an example of an option to delay (see table 1). According to the traditional NPV-rule, this is a valuable project which should be implemented immediately. For, the NPV amounts $\sum_{t=1}^{15} \frac{120}{(1{,}10)^t} - 800$ or 112.73 million EUR. Does real option analysis yield a different result? The different option parameters take the following values: V = 912.73, I = 800, T-t = 1 (decision is postponed for one year), σ = 0.10 and r = 0.10. If we enter these data in the Black-Scholes model of equations [1] to [3] we obtain a real option value of 189.14 million EUR[8]. Because the real option value (measuring the delay of the investment) exceeds the NPV-value (measuring the immediate investment), it is better for the firm to postpone the decision with one year instead of investing immediately.

2.Criminal behavior as a real option

Traditionally, criminal behavior is analyzed within an expected utility framework, which states that a criminal will commit an offense if the expected profits exceed the expected costs

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[8] Equations [2] and [3] are equal to $d_1 = \frac{\ln\left(\frac{912.73}{800}\right) + \left[0{,}1 + \frac{1}{2}(0{,}1)^2\right] \cdot 1}{(0{,}1)\sqrt{1}} = 2{,}37 \quad and \quad d_2 = 2{,}27$, respectively. Equation [1] is thus equal to $C = 912.73 \cdot (0.9911) - 800 \cdot e^{-0.1 \cdot 1} \cdot (0.9884) = 189.14$.

(Cooter and Ulen, 2000). The expected profits are the gains (Y) that result from the offense. The expected costs are the product of the probability of conviction (*p*) and the level of punishment (*f*)[9]. The expected net-gain is thus $Y - p.f$. This conventional decision rule is very simple[10]:

- if Y > p.f : a crime is committed [4]
- if Y < p.f : no crime is committed

This traditional calculation of the expected net-gain is therefore a now-or-never decision. It ignores aspects of uncertainty by which a crime can become profitable after all. The conventional approach cannot deal with the simultaneous existence of three phenomena: uncertainty, irreversibility of the crime and some freedom of choice on the timing of the crime. Combining irreversibility with the existence of uncertainty over the future behavior of variables that affect the value of the crime leads to the following intuitive reasoning. Suppose there is some leeway in delaying the crime until more information about the uncertain future becomes available. It may then be optimal to wait some time before committing the crime. It is clear that waiting to commit the crime implies risks (e.g. entry of other criminals) and foregone profits, but it may prevent from being trapped in an irreversible crime, which turns out to be very costly when the adverse future materializes, i.e. being caught.

A crime that satisfies these three characteristics is best treated analogous to holding a financial call-option. For some specific time period, a criminal has the possibility, but not the obligation, to pay a certain 'price' in return for an asset that has some value. When the criminal decision is made, the option is exercised, which is an irreversible decision.

The concept of a crime as a real option is visualized in figure 1. As long as the benefits from the crime are smaller than the costs of the crime, the criminal will not exercise his criminal option (left side of figure 1 – criminal option is out-of-the-money)[11]. Once the benefits of the crime

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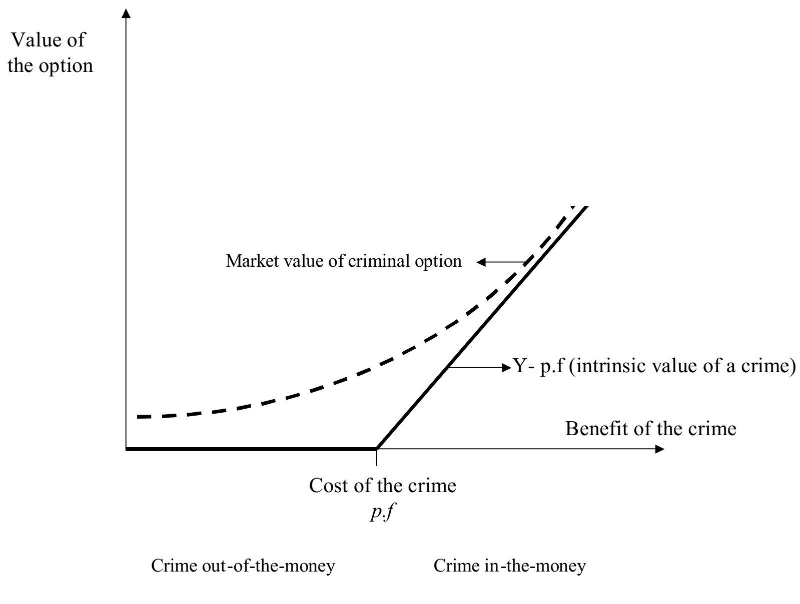
[9] More formally, a criminal will maximize the following expected utility function (Becker, 1968, 177): $EU = pU(Y - f) + (1 - p)U(Y)$. See also chapter seven in Engelen (2002) on using state-dependent expected utility models and rank-dependent expected utility models.

[10] This approach is similar to the classical NPV-rule in investment analysis: $NPV = \sum_{t=1}^{N} \frac{FOCF_t}{(1+k)^t} - I_0$, with N = lifetime of the investment project; k = expected return on a project with identical risk; I_0 = initial investment expenditure and $FOCF_t$ = free operating cash flows. The traditional investment rule is: NPV > 0: invest *now* or NPV < 0: invest *never*.

[11] An call-option is said to be out-of-the-money when the stock price is lower than the exercise price and in-the-money when the opposite is true.

exceed the costs of the crime, a criminal can consider to exercise the criminal option and thus commit the crime (right side of figure 1 – criminal option is in-the-money). If the criminal exercises its option, he will receive the intrinsic value of the option $Y - p.f.$ So, by exercising the option, he loses the time value of the criminal option[12]. Again, an analogy with financial options can be made. For instance, on March 1st 2001 a call-option on the stock *ABN AMRO* on the *Euronext Option Market Amsterdam* with expiration date October 2001 and exercise price 22.68 EUR is traded at 2.80 EUR. At that moment, ABN AMRO was trading at 23.93 EUR. If an investor buys this option and would have exercised the call-option on the same day, he would receive (S) 23.93 EUR minus (X) 22.68 EUR or 1.25 EUR. This is the intrinsic value of the call-option. Compared to the price of the option (2.80 EUR) he would suffer a loss of 1.55 EUR in this way. However, this sum is the time value of the option and can be considered as a premium the investor has paid for the flexibility to wait and exercise the option at a later date when the stock price increased further.

Figure 1. Criminal behavior as a real option



Like a financial option, the criminal option itself has some (non-negative) value, a.o. because of the uncertainty over the future value of the crime. As a consequence, option pricing theory can be used to 'price' criminal decisions and decide on optimal timing of exercise. Again, referring to the analogy between the parameters of financial and real options and equations [1] to [3], the value drivers of a crime in real options terms are: Y = benefits of the crime, p.f = costs of the

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[12] The time value of an option is the difference between the market value of the option (dashed line in figure 1) and the intrinsic value (solid line in figure 1).

crime, T-t = time to expiration, r_F = risk-free interest rate, σ = volatility of expected benefits of the crime and δ = opportunity cost or the value lost during duration of the option. Table 2 summarizes this analogy between financial, real and criminal options. It is clear that conventional economic analysis of crime only takes two parameters into account, Y and p.f (see upper panel of figure 2), while real or criminal option analysis takes six variables into account (see lower panel of figure 2). Besides the benefits and the costs of the crime, the option model takes into account the time to expiration, the risk-free interest rate, the volatility of the expected benefits of the crime and the opportunity cost by not committing the crime immediately.

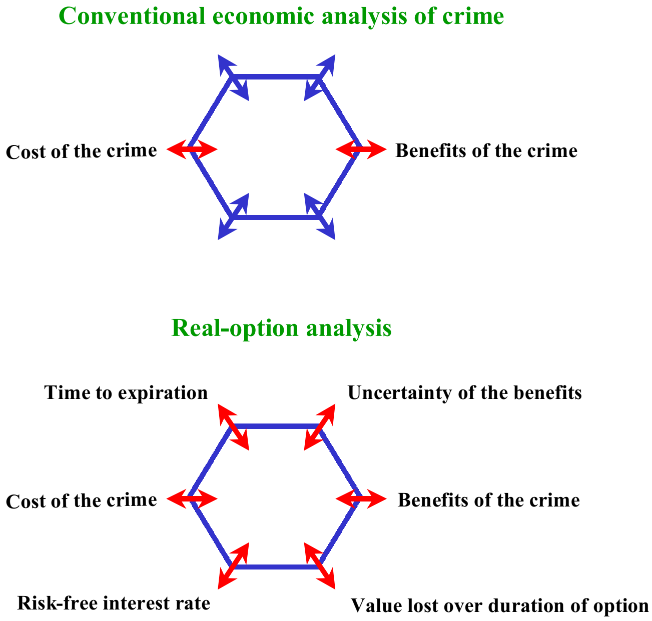
Table 2.The analogy between criminal, real and financial options

Financial option		Real option		Criminal option	
Stock price	S	Present value of future FOCFs	V	Benefits of the crime	Y
Exercise price	X	Investment expenditure	I	Costs of the crime	p.f
Stock return volatility	σ	Project return volatility	σ	Volatility of return of crime	σ
Time to expiration	T-t	Duration of real option	T-t	Duration of criminal option	T-t
Risk-free interest rate	r_F	Risk-free interest rate	r_F	Risk-free interest rate	r_F
Dividendyield	δ	Opportunity cost	δ	Opportunity cost	δ

Note the one important feature of option pricing models is the concept of risk neutral valuation. This means that the risk preferences of investors, or in this case criminals, don't matter when pricing derivatives. Put differently, it means that the valuation only involves variables that are unaffected by the risk preferences of investors or criminals. As Hull (2000, 249) points out, the solutions of an option pricing model (using risk-neutral valuation) are valid in *all* worlds, not just those where investors (or criminals) are risk neutral[13].

[13] However, Rubinstein (1976) shows that option pricing formulas can also be derived under risk aversion.

Figure 2. Real-option analysis versus conventional economic analysis of crime



The difference between conventional economic analysis of crime and criminal option analysis becomes more clear when it is analyzed under which conditions both models lead to similar conclusions and under which conditions to different conclusions. When there is no uncertainty about the proceeds of the crime (technically, this means the volatility, σ, amounts to zero) or when the decision to commit the crime can no longer be deferred (technically, T-t = 0) conventional economic analysis of crime and criminal option analysis yield similar conclusions. The net-gain according to the conventional economic analysis of crime yields *Y–p.f*. If Y is larger than the cost (*p.f*) a crime is committed. According to the criminal option model, the option only takes the intrinsic value[14]. So the value of the criminal option is *Y–p.f* or 0, whichever is the highest[15]. If the option takes a positive value, it will be exercised and, again, the crime will be committed. As can be seen, both models lead to similar conclusions.

When there is some uncertainty about the proceeds of the crime (technically, this means the volatility takes a positive number) or when the decision to commit the crime can be deferred for some time (technically, T-t > 0) conventional economic analysis of crime and criminal option analysis can lead to different conclusions. For instance, it may very well be the case that a crime is not profitable according to the conventional economic analysis of crime. Based

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[14] If T-t = 0 or σ = 0, then the option has no time value.
[15] The reader may notice that the payoff in the case of the conventional economic analysis of crime is also zero when the crime is not committed.

on this model, the crime will never be committed. However, the criminal option may show that the crime can become profitable in the near future. So, the conclusion of the option model is not 'never commit this crime' but it simply postpones the criminal decision. If in the near future the crime becomes profitable, the option may be exercised. In the other case, the option expires and no crime will be committed. Another example is the case of a profitable crime according to the conventional economic analysis of crime. In this case, the crime will be committed immediately. Again, a criminal option analysis may show that, although the crime is already profitable, it is economically more valuable to wait (see section 6.4).

2.Insider trading as a criminal option

While the previous section showed that all crimes can be seen as real options, i.e. they confer the possibility but not the obligation to commit a crime in the future, the current section applies this model to illegal insider trading[16]. Insider trading is trading based on non-public information about a certain firm-specific event that can influence the price of a security. Suppose corporate insiders possess private information that is, of course, not known to other market participants, e.g. the development of a significant new product. By the development of this new product, the corporate insiders created valuable information for investors. The mere possession of the private information gives the insiders the option to trade based on this information. This crime is an option because the insiders have the possibility, but not the obligation, to trade on inside information.

Analogously to the general criminal option model in table 2 we can complete all six value drivers or parameters of the option model. The first parameter (Y) includes the benefits the criminal will receive from insider trading. These are the realized capital gains on the traded shares. Analogously to the conventional economic analysis of crime, the cost of insider

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[16] This chapter focuses on illegal insider trading. However, not all trading by insiders is prohibited or illegal. Legal insider trading refers to the transactions by corporate insiders that have to be reported to the supervising authorities. For instance, in the U.S. 'officers', 'directors' and beneficial owners of more than ten percent of any class of stock to disclose their fraction of share ownership and their transactions in shares of their company (Section 16a-3 (a) Securities Exchange Act of 1934). Within ten days of obtaining their insider status, insiders have to disclose their initial fraction of ownership in the company via a Form 3. Subsequent changes in their fraction of ownership have to be disclosed via a Form 4 by the tenth day of the month following on the month of the transaction. Moreover, insiders have to disclose their fraction of ownership within 45 days after the fiscal year-end via a Form 5. The SEC distributes these notifications to the investment public through the publication 'Official Summary of Security Transactions and Holdings'. This legal insider trading has to be distinguished from illegal insider trading prohibited by section 10(b) of the Securities and Exchange Act of 1934 and SEC rule 10b-5.

trading is the product of the probability of conviction for committing insider trading (*p*) and the level of punishment for committing insider trading (*f*). The duration of the criminal option (T-t) will be rather short in the case of insider trading, varying from a couple of hours, days or maximum some weeks. It is the time between the possession of the private information and the official disclosure of that information through a public announcement. It is clear that the expiration date of this option is the moment on which a public announcement is made. On that moment the corporate insider does no longer possess private information. The volatility is the standard deviation on the return of the crime. The final parameter (δ) is the opportunity cost by not exercising the option immediately. This can be compared to the dividend in case of financial options. As long as the holder of a call-option doesn't execute the option, he is not entitled to the dividends of the underlying share. The missing of this dividend reduces the value of the option. In the same way, δ is the value lost during the duration of the criminal option. For, by not trading on the inside information immediately, the profits based on insider trading may decline. This is the case if parts of the private information leak to the market before the insider exercises his option. These information leakages may cause the stock price to rise (in case of good news). In this way, the potential capital gains of the insider will be smaller. Summarizing, these parameters can be labelled as: Y = capital gains of insider trading, p.f = cost of insider trading, T-t = time during which insider possesses privileged information, r_F = risk-free interest rate, σ = volatility of expected benefits of insider trading and δ = value lost during duration of the option by not trading immediately.

Let's illustrate the use of option-pricing model to analyze criminal behavior in case of insider trading by a stylized example. Suppose a corporate insider possesses some private price-sensitive information. Currently, the shares of his company are traded at 37.5 EUR. Between the moments of the possession of this private information and the public announcement lies a period of one month. Furthermore, suppose the insider buys 400 shares of his own company based on the inside information. He expects the new equilibrium price to be 50 EUR once the news has publicly been disclosed (this is equal to a price increase of 33%). The expected level of punishment (*f*) equals 47,500 EUR, while the probability of conviction (*p*) is 10%. The risk-free interest rate is 5% and the volatility of the return of the crime is 40%. Finally, suppose the opportunity cost is equal to 1%. Should the corporate insider trade on his inside information immediately or should he postpone his decision?

According to the conventional cost benefit approach he will commit the crime immediately because the expected net-gain, Y – p.f, is positive and equal to 5000 EUR minus 4750 EUR or 250 EUR. Next, the criminal option model is applied to this situation. Should the insider trade immediately on the information or better wait? Using a Black-Scholes model to price this criminal option, the different parameters can be calculated as follows. The expected capital gains are 400 shares times (50 – 37.5) EUR or 5,000 EUR. The expected costs of this crime are (0.10)*47,500 EUR or 4,750 EUR. The duration of the option is one month or 0.0833 years, the volatility is 0.40, the risk-free rate is 0.05 and δ is 0.01. Entering this data in the option-pricing formula yield an option value, C_0, of 380.95 EUR[17]. As can be seen, the option value (380.95) is higher than the conventional cost benefit value (250). Therefore, according to the criminal option model, the corporate insider should wait before deciding when to commit the crime.

Alternatively, calculations were made using a value of 50,000 EUR instead of 47,500 EUR for the level of punishment (*f*). In this case the conventional cost benefit approach yields a value of 0 EUR. According to this approach the crime will never be committed. If one calculates the option value of this crime, the option is worth 237.80 EUR. Therefore, the decision according to the option model is to wait. It may very well be the case that the criminal option becomes valuable after all.

The next two sections analyze insider trading as a real option in more detail. Section four examines this criminal option from the point of view of the criminal, while section five is the point of view of the legislator. How can insider trading be restricted based on the findings of an option model?

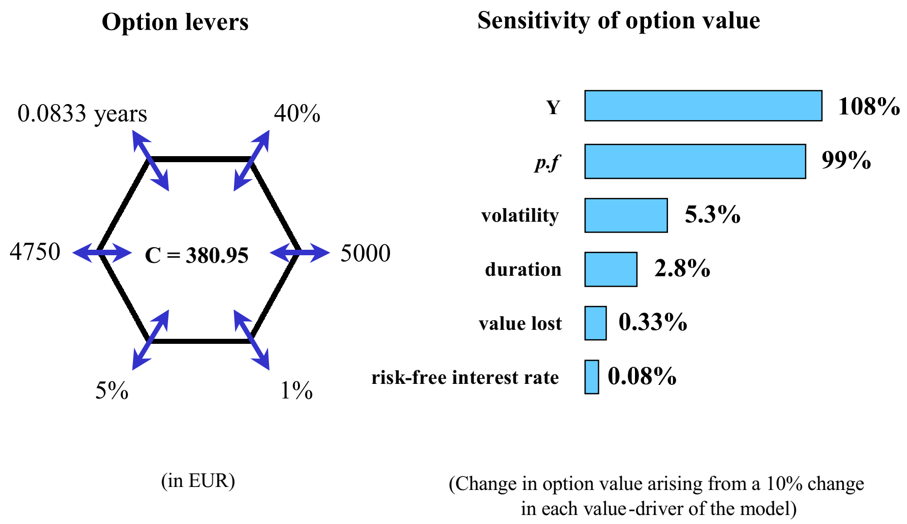
4.The point of view of the criminal – pro-active management of the criminal option by the criminal

If a criminal uses a criminal option model to determine whether (or when) to commit a crime, this approach can have major implications for the criminal enforcement policy in general and the restriction of insider trading in particular (see section five). As the above sections showed, all criminal decisions can in fact be seen as real options, i.e. they confer the possibility but not

[17] Referring to equation [1] to [3], d_1 and d_2 amount 0.5300 and 0.4145 respectively and $N(d_1)$ and $N(d_2)$ amount 0.7019 and 0.6607 respectively.

the obligation to commit a crime in the future. Such criminal options can be valued using an option valuation model, e.g. the Black-Scholes model. As such, this model can be used to determine which parameters or value-drivers of the model will cause the value of the criminal option to raise.

Figure 3. Sensitivity of the different value-drivers of the criminal option model



Let's consider again the example of the previous section. Once, the option value is calculated, a sensitivity analysis can be carried out on this base case. Figure 3 illustrates this exercise. The left panel of this figure gives the different value-drivers of the base case, while the right panel gives the percentage change in the option value arising from a 10% change in each value-driver of the model. For instance, if the expected costs from the crime decrease by 10%, the option value doubles in value (+99%). Or, if the volatility increases by 10%, then the option value increases 5.3%. As can be seen, the criminal option in this example is most sensitive for a change in the benefits and costs of the crime (+108%). The value lost over the duration of the option and the risk-free interest rate have little influence (+0.33% and +0.08% respectively). However, other criminal options may very well yield different conclusions about the most important value-drivers.

A more formal analysis of the sensitivity of the option value to the different value-drivers is the calculation of what is commonly known as the 'Greeks' (Wilmott, 1998). For, as explained below, understanding how a change in one of the value-drivers affects the criminal option value is very important for the criminal enforcement policy. To quantify these dynamics, the partial[18] derivatives of the option value with respect to the six parameters must be calculated. These derivatives are summarized in table 3 (See appendix 1 for a detailed derivation of the Greeks).

Table 3.The Greeks – derivatives of a criminal option

Sensitivity of C to:	Name and symbol		Formula
Benefits of the crime (Y)	Delta	Δ	$e^{-\delta(T-t)} N(d_1)$
Volatility (σ)	Vega	ν	$Y\sqrt{T-t}\, n(d_1) e^{-\delta(T-t)}$
Duration (T-t)	Theta	Θ	$\frac{-Y\, n(d_1)\sigma\, e^{-\delta(T-t)}}{2\sqrt{T-t}} - r\,(p.f)e^{-r(T-t)} N(d_2) + \delta\, Y\, N(d_1) e^{-\delta(T-t)}$
Risk-free interest rate (r)	Rho	ρ	$(p.f)(T-t)e^{-r(T-t)} N(d_2)$
Cost of the crime (p.f)	Psi	Ψ	$-e^{-r(T-t)} N(d_2)$
Opportunity cost (δ)	Ksi	Ξ	$-Y(T-t)e^{-\delta(T-t)} N(d_1)$

Note: The value $n(d_1)$ is the derivative of the standard normal distribution function with respect to d_1

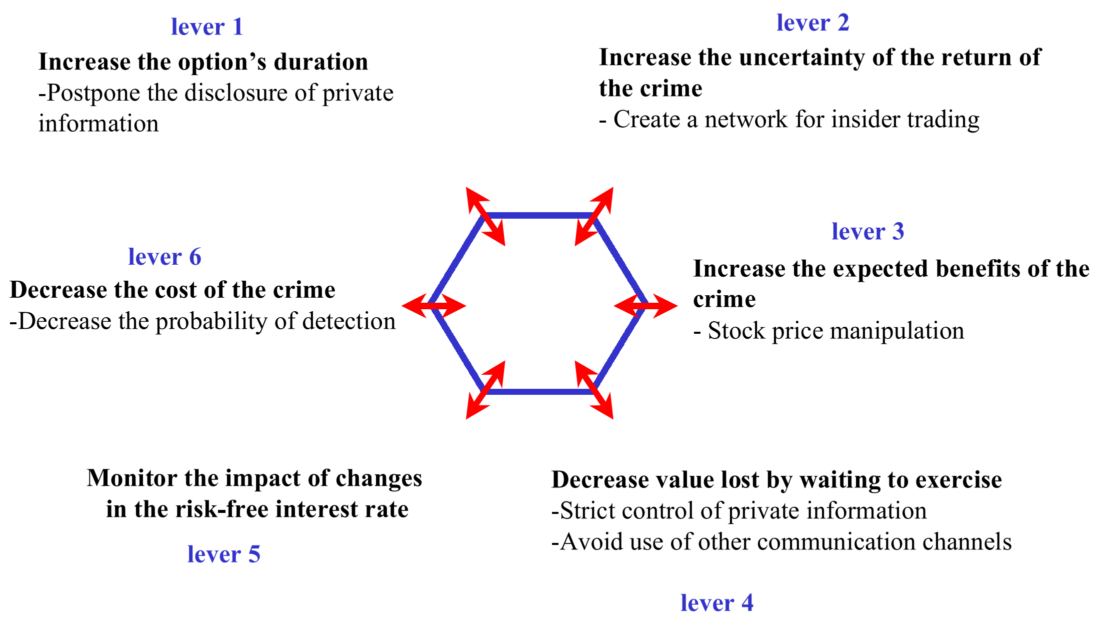
Delta is the partial derivative of the option value (C) with regard to the benefits of the crime (Y). This derivative is always greater than zero, meaning that the higher the benefits of the crime, the more valuable the criminal option. The derivative with regard to volatility, vega, also has a positive sign. This is a consequence of the asymmetric payoff profile of an option. A higher volatility increases the upside potential of the option, while at the same time the downside loss is limited. For, if the option expires out-of-the-money, the option will not be exercised. Theta is always negative. The shorter the time to expiration, the less valuable the criminal option. Rho is the partial derivative of C with regard to the risk-free interest rate. Finally, the derivative with regard to the cost of the crime (*p.f*) is denoted as psi. The higher the cost of the crime, the less valuable the criminal option.

By calculating the Greeks and simulating the sensitivity of the different value-drivers of his real option, a criminal can actively manage his (portfolio of) real options. Instead of passively

[18] While we allow one parameter to change, we hold all other parameters constant.

monitoring his criminal option, an active management of the six value-drivers can increase the option value. In order to maximize his criminal option value, the criminal can pursue different strategies (see figure 4). He can, for instance, increase the duration of the criminal option by trying to postpone the disclosure of the private information (lever 1), increase the expected benefits of the crime by simultaneously manipulating stock prices (lever 3) or try to avoid information leakages to minimize the opportunity costs (lever 4).

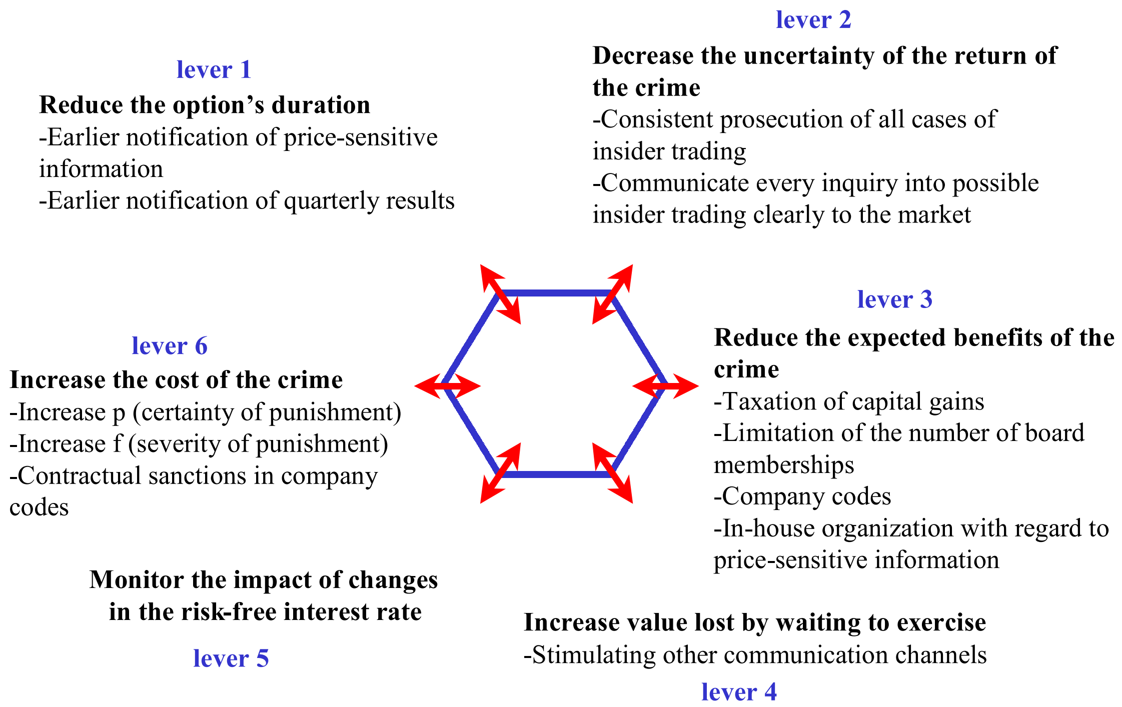
Figure 4. Active management of criminal behavior by the criminal



4.The point of view of the legislator – criminal enforcement policy

A thorough examination of the sensitivity of the different value-drivers of a criminal option is also very important in order to establish an adequate criminal enforcement policy. Once the most valuable parameters of a particular kind of criminal options are identified, the enforcement policy can occur more efficiently by focusing on these parameters. Just as criminals can actively manage their (portfolio of) real options, the criminal legislator and prosecuting authorities can actively manage the enforcement of criminal options. Instead of passively monitoring crimes, an active enforcement policy based on the six value-drivers of criminal options can be pursued. Whilst criminals can develop strategies to increase the real option value, the point of view of the enforcement policy is just the opposite, i.e. develop an adequate policy to decrease the value of the criminal option by using one of the six parameters.

Figure 5. Active management of criminal behavior by the legislator



In order to minimize the criminal option value, the legislator and supervising authorities must pull one or more levers as visualized in figure 5. Lever one consists of reducing the duration of the criminal option. This can be obtained by an earlier notification of price-sensitive information, preferably during the trading hours of the stock exchange (Engelen and Kabir, 2001). In this way, the time span for the insider to act on his private information is reduced. Furthermore, an earlier notification of quarterly results and semi-annual reports can also reduce the time to expiration. For instance, Engelen (1999) points out that Belgian companies on the Euronext Brussels distribute their semi-annual reports more than 70 days after the closing of the first half-year. When this term is compared to the disclosure policy on e.g. NASDAQ, which uses a publication term of 45 days, an earlier notification can reduce the option's duration significantly. This can be measured analytically by decreasing the time to maturity (T-t) from e.g. 70 days to 45 days.

Lever two is the decrease of the uncertainty of the expected return of the crime. In this case, the corporate insider will especially be sensitive to a reduced uncertainty concerning the expected cost component. Presently, there is considerable uncertainty with respect to the detection and prosecution of insider trading. A consistent prosecution of all cases of insider trading can be an efficient signal to potential criminals. In this way, the criminal is confronted with less uncertainty about the enforcement policy. Also, a clear and consistent communication of every inquiry into possible cases of insider trading to the market can add to reduce the uncertainty. Supervising authorities have no impact on the uncertainty of the expected benefits component of the crime. For, it is the market that determines the new equilibrium stock price once the price sensitive information has been disclosed.

A reduction of the expected benefits of the crime is a third lever of an adequate enforcement policy. An efficient way to reduce the expected benefits of insider trading, i.e. the capital gains, is taxation. A general capital gains tax rate of e.g. 30% would also reduce the expected benefits of insider trading by the same percentage[19]. Another measure can be the limitation of the number of board functions one can accumulate. In this way, the potential number of cases of private information will be limited. Furthermore, supervising authorities can impose company codes as an admission requirement to listing on the stock exchange. For instance, the former EASDAQ-code obliged companies to use closed periods. According to the EASDAQ Dealing Code, directors or senior executive were prohibited from dealing (a) in the period of two months immediately preceding the preliminary announcement of the issuer's annual results or, if shorter, the period from the relevant financial year end up to and including the time of the announcement, (b) in the period of one month immediately preceding the preliminary announcement of the issuer's quarterly results, or, if shorter, the period from the relevant quarter end up to and including the time of the announcement, or (c) in the period of fifteen days immediately preceding the announcement by the issuer of any price sensitive information. Next, supervising authorities can lay down rules how listed companies should deal with the in-house organization of price-sensitive information. Companies can limit the free dissemination of price sensitive information within the company by e.g. Chinese walls, the need-to-know basis or the use of code names (Cruyt, 1990). Chinese walls are commonly used in financial institutions where there is a strict separation

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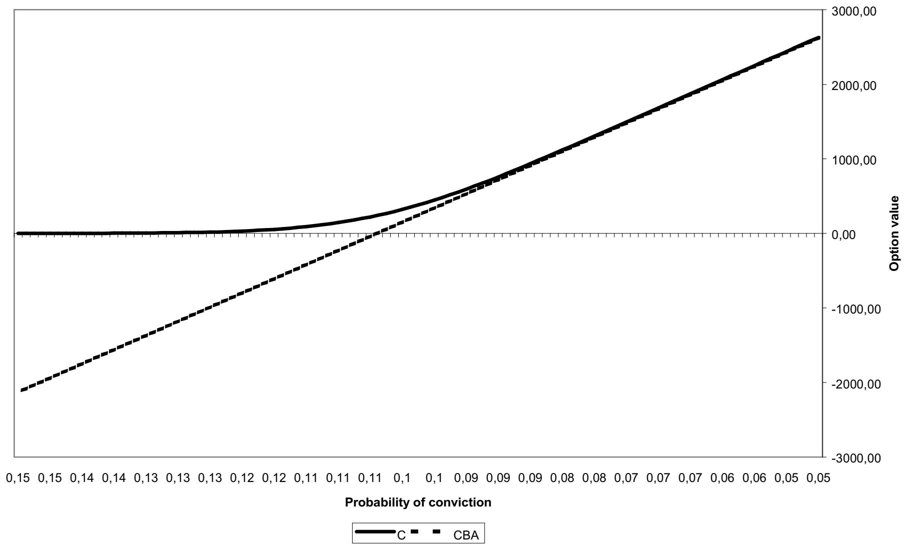
[19] We only propose this measure from the point of view of restricting insider trading. Other consequences are left out of consideration.

between the credit department and the investment department. The need-to-know basis implies that within one business unit not all employees or staff members have to be acquainted with a price sensitive project of a co-worker. Companies can also use code names for price sensitive projects (e.g. an acquisition).

The fourth value-driver of a criminal option is the value lost by waiting to exercise. If supervising authorities can increase this opportunity cost over the duration of the option, they have another lever to restrict insider trading. This can be achieved by stimulating other communication channels with the market that can signal the private information to the market. Examples are the reporting of changes in the ownership of securities of their own company by officers and directors or the announcement of share repurchase programs. The information value of share repurchase programs is strongly empirically supported (Vermaelen, 1981, Dann, 1981, Asquith and Mullins, 1986, Comment and Jarrell, 1991, Bagwell, 1992 and Ikenberry, Lakonishok and Vermaelen, 1995). This is a very reliable signal to communicate private information to the market. In the States, corporate insiders must notify their transactions in securities of their company. This legal insider trading is reported in the *'Official Summary of Security Transactions and Holdings'*. Again, the superior information value of these notifications is empirically supported (Jaffe, 1974, Finnerty, 1976 or Givoly and Palmon, 1985). Other signals are the use of financial analysts (Fischel, 1984), or the amount of debt of a listed company (Ross, 1977). Through different communication channels information will reach the market, moving the stock price closer to its fundamental value and therefore affect the potential benefit of illegal insider trading.

The enforcement policy has no impact on the level of the risk-free interest rate. This parameter is exogenous and its impact can only passively be monitored.

Figure 6. The impact of a change in p: conventional economic analysis of crime vs. option model



The final lever of the enforcement policy is the increase of the cost of the crime. This is analogous to the conventional economic analysis of crime, either an increase of the certainty of punishment (*p*) or an increase of the level op punishment (*f*). These solutions are explained in detail in Engelen (1997, 2002). The only difference between a criminal option model and the conventional economic model of crime is the fact that a change in one of these parameters will not have the exact impact on the NPV or on the option value. This can be seen when we compare the slope coefficient of the NPV-model (represented by the dashed line in figure 6) with the derivative of C with respect to exercise price (Psi) (option model is represented by the solid line in figure 6). The slope coefficient is equal to –1, while psi can be calculated as $-e^{-r(T-t)}\,N(d_2)$.

6.Extensions to the model

While the previous sections explained the criminal option approach, this section introduces some extensions to the base model, such as the use of American options, the problem of uncertain exercise prices and the issue of the optimal timing of a crime.

6.1.The probability of conviction

Because the cost of the crime is composed of the probability of conviction (*p*) and the level of punishment (*f*), one can divide this parameter into two components, being *p* and *f*. Suppose the level of punishment is fixed. One can then analyze the impact of a change in the probability of conviction (p) on the criminal option value. Figure 7 illustrates this on the above example. In the base case, the probability of punishment was 10%. If the enforcement policy could be optimalized, causing this probability to rise, the option value decreases drastically. The sensitivity of changes in the option value with regard to p can be formally expressed as equation [5][20]:

KAPPA: $$\frac{\partial C}{\partial p} = -f \cdot e^{-r(T-t)} \cdot N(d_2) < 0 \qquad [5]$$

Figure 7. The option value with respect to the probability of conviction (p)

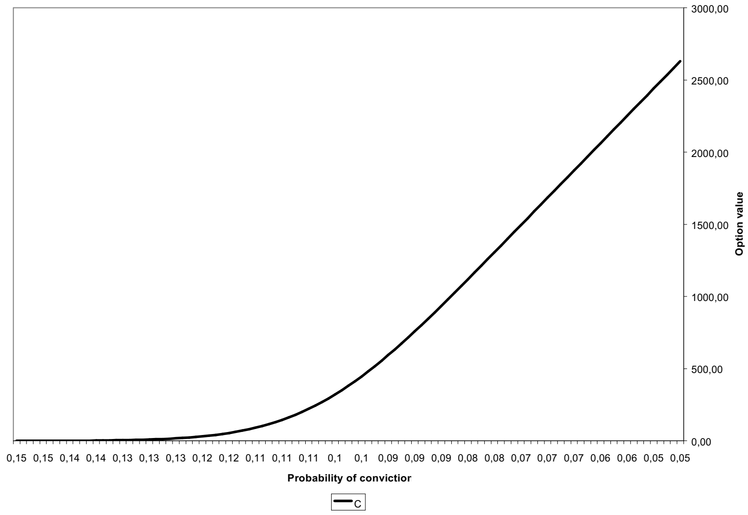
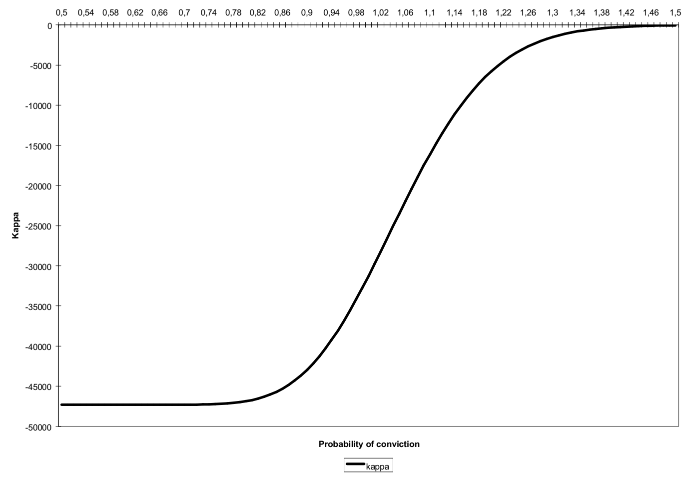


Figure 8. Variation of kappa with the probability of conviction for a criminal option



6.2.American options

While the above sections analyzed criminal options, the implicit assumption was made that those options could only be exercised on the expiration date (European option style).

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[20] See also figure 8.

However, it is more realistic to assume that criminals can exercise their criminal option during the whole time to maturity (American option type). It can be demonstrated that the value of an American option is equal to or higher than the value of a European option (Gibson, 1991). However, if the dividend yield or the opportunity cost (δ) is zero, an American option will never be exercised prior to its expiration date. In this case its value is equal to the value of a European option. Furthermore, as Hull (2000, 260) demonstrates, the early exercise of an American option prior to its expiration date is possible, if the dividend yield (or the opportunity cost, δ) is equal to or above the risk-free interest rate. In this case, the American option can be valued using a binominal option pricing model (Elton and Gruber (1995, 582).

6.3.Uncertain exercise price

The above option pricing models assume a fixed exercise price. However, in case of criminal options the exercise price can be uncertain sometimes. For, the exercise is composed of the probability of conviction (*p*) and the level of punishment (*f*). Although the level of punishment is rather fixed in the short term, the probability of conviction is uncertain. If one assumes the exercise price to be fixed in such a case, the Black-Scholes model would value the criminal option incorrectly. In this case an option pricing model that incorporates the uncertain exercise price has to be used, as is the case with the models of Fisher (1978, p.172) and Margrabe (1978, p.1979). According to this model the value of the criminal option depends on the standard deviations of the benefits of the crime and of the exercise price and the correlation between them. The Fisher and Margrabe model formulated in criminal option terms is:

$C = Y\,N(d_1) - (p.f)\,N(d_2)$, where [6]

$$d_1 = \frac{\ln\left(\frac{Y}{p.f}\right) + \left(\frac{\hat{\sigma}^2}{2}\right)(T-t)}{\hat{\sigma}\sqrt{T-t}} \qquad [7]$$

$$d_2 = \frac{\ln\left(\frac{Y}{p.f}\right) - \left(\frac{\hat{\sigma}^2}{2}\right)(T-t)}{\hat{\sigma}\sqrt{T-t}} \qquad [8]$$

, with:

$$\hat{\sigma}^2 = \sigma_Y^2 - 2\rho_{Yx}\sigma_Y\sigma_x + \sigma_x^2 \qquad [9]$$

Y = capital gains of insider trading

$p.f$ = cost of insider trading

$T-t$ = time during which insider possesses privileged information

r_F = risk-free interest rate

σ_Y = volatility of expected benefits of insider trading

σ_x = volatility of exercise price

ρ_{Yx} = correlation coefficient between the expected benefits and the exercise price

$N(.)$ = cumulative standard normal density function

Compared to the standard Black-Scholes model, this model requires the input of two additional parameters, i.e. the standard deviation of the exercise price and the correlation between the expected benefits of the crime and the exercise price. Suppose we apply this model to the above example. This means that all variables take the same value as in the above example, except for the two additional variables, which we assume to be $\sigma_x = 0.50$ and $\rho_{Yx} = 0.80$. If we input this data in this model, the option value amounts 322.05, compared to the Black-Scholes value of 380.95 (see supra).

6.4.Optimal timing of a crime

One of the consequences of viewing the criminal decision as exercising an option is the problem *when* to commit a crime. This can be illustrated most simply by referring to the conventional economic analysis of crime: the direct payoff from committing the crime is given by $Y\text{-}p.f$ (compare to the traditional NPV-criterion). When this payoff is positive ($Y\text{-}p.f > 0$)[21], it is worthwhile to commit the crime. However, once the crime is made, the option is gone. Therefore, we can apply the general category of options to delay[22] to this criminal option to analyze this situation. Should the criminal, in this case the corporate insider, commit the crime now, or wait until more information is available so that his criminal decision can be made under less uncertainty. For, by trading on inside information, the option expires. So, the value of the option today, C_0, must be considered as an opportunity cost of committing the crime

[21] See also equation [4].
[22] See also table 1.

immediately, and hence, must be added to the cost of the crime, *p.f.* Hence the optimal crime criterion is modified into[23]:

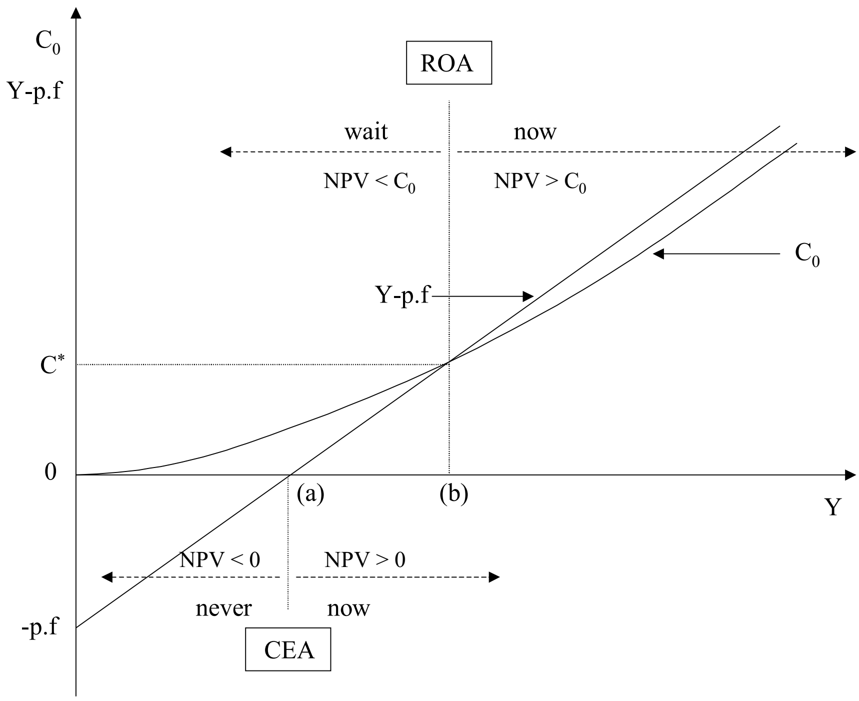
$Y < p.f + C_0$ or $NPV = Y - p.f < C_0$: wait to commit the crime [10]

$Y > p.f + C_0$ or $NPV = Y - p.f > C_0$: commit the crime

Another way of indicating the same criterion is stating that the value of the crime, Y, must exceed the cost of the crime, *p.f*, by at least the value of the option, C_0, in order to decide to commit the crime now. This minimum-acceptable crime value can be called the 'threshold value' of the crime, in the following denoted as Y^*. In option pricing jargon, the option is said to be 'out of the money' in the case of 'waiting' and 'in the money' when the underlying value of the crime, Y, exceeds the option value plus the cost of the crime. As such, the basic criminal decision to take is not *whether or not* to commit the crime (as indicated by the conventional economic analysis of crime), but rather *when* to commit the crime, i.e. determining the optimal moment of exercising the criminal option. This intuitive reasoning is graphically represented in figure 9. The lower panel of this figure shows the conventional cost benefit approach to crimes. According to the NPV-rule the crime will be committed when the underlying value, Y, amounts the value indicated by point (a) in figure 9. However, the option model shows that the optimal timing of exercising the criminal option is point (b) instead of point (a). If Y takes the value in point (b), the upper panel of the figure shows that the optimal timing to commit the crime has been reached. For, in point (b) Y has reached its threshold value Y^*. See Dixit & Pindyck (1994) on numerical methods to calculate this threshold value.

[23] This is similar to investment decisions in finance: if a project can be delayed, real option analysis shows that the project will be implemented immediately if $NPV > C_0$ and delayed if $NPV < C_0$. Once, the option expires the final decision to implement the project is taken based on the information available on that moment in time. As such, the conventional NPV-rule fails to capture this flexibility that has clearly value to a company.

Figure 9. Crimes from the point of view of real option analysis (ROA) versus the conventional economic analysis of crime (CEA)



7.Conclusions

Compared to the conventional economic analysis of crime, this paper presented a complete new model to analyze criminal behavior based on the concept of real options. It was shown that all criminal decisions can be analyzed as real options, because they confer the possibility but not the obligation to commit a crime in the future. The differences between traditional criminal models and criminal option analysis were analyzed, in particular under which conditions both models lead to similar conclusions and under which conditions to different conclusions. As such it is possible that a crime is not profitable according to the traditional models and will never be committed, while the criminal option may show that the crime can become profitable in the near future. So, the conclusion of the option model is not 'never commit this crime' but it simply postpones the criminal decision until the criminal decision can be made under less uncertainty.

Next, this model is applied to insider trading. The mere possession of the private information gives insiders the option to trade based on this information. This crime is an option because the insiders have the possibility, but not the obligation, to trade on inside information. It was shown that the criminal option approach is a richer model compared to conventional economic models of crime. While the conventional models only incorporate two variables, being the benefits of the crime and the costs of the crime, criminal option models take into account four additional variables, such as the time to expiration, the risk-free interest rate, the volatility of the return of the crime and the opportunity cost by not committing the crime immediately. Based on these six value-drivers of criminal options, which we labeled as the Greeks, an active management strategy can be developed for both the criminal as for the legislator.

Instead of passively monitoring his criminal option, the criminal can pursue an active management of his criminal option by enhancing the value of one of the six levers of the option. Among the different strategies, he can increase the duration of the criminal option by trying to postpone the disclosure of the private information, he can increase the expected benefits of the crime by simultaneously manipulating stock prices or he can minimize the opportunity costs by limiting the possibilities of information leakages to the market.

Finally, this chapter examined criminal options from the point of view of the legislator. How can insider trading be restricted based on the findings of an option model? Again, an active management of enforcement policy of insider trading involves six levers. For each lever, an active management strategy by supervisory authorities can reduce the value of the criminal option. First, the time to expiration of the option can be reduced by earlier notifications of price-sensitive information and by earlier notifications of quarterly results. Second, the uncertainty of the return of the crime can be reduced by a consistent prosecution (and communication) of all cases of insider trading. Third, a reduction of the expected benefits of the crime includes a taxation of capital gains, a limitation of the number of board functions, the use of company codes and an in-house organization with respect to price-sensitive information. Fourth, by stimulating other communication and signaling channels, the opportunity cost of the option increases making the criminal option less valuable. Finally, the costs of the crime can be increased by raising the severity of punishment or the probability of conviction.

Because this paper is only an introduction of criminal option models, future research has to elaborate the analysis, to refine some of the new insights and to make the model more operational. We already suggested some extensions of the model in section five by including

American options, by focusing on uncertain exercise prices and by analyzing the optimal timing of a crime. Further research should focus on the optimal timing problem as well as on the appropriate valuation models for different categories of criminal options. Nevertheless, it is a very promising new way of looking at criminal behavior and enforcement policy.

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Appendix 1. Partial derivatives of the Black-Scholes formula

$C = S\,e^{-\delta(T-t)}\,N(d_1) - X\,e^{-r_c(T-t)}\,N(d_2)$, where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_c - \delta + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{X}\right) + \left(r_c - \delta - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$, with:

r_c = continuous risk-free interest rate, T-t = time to expiration, N(d) = cumulative normal probability density function, σ = standard deviation of the stock return, δ = dividend yield, S = current stock price, and X = exercise price.

a. Partial derivative with regard to S:

$$\begin{aligned}
\frac{\partial C}{\partial S} &= e^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial S} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial S} \\
&= e^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial S} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial S} \\
&= e^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial S} - Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial S} \\
&= e^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial S} - Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_1}{\partial S} \\
&= e^{-\delta(T-t)}N(d_1) + n(d_1)\frac{\partial d_1}{\partial S}\left[Se^{-\delta(T-t)} - \frac{X}{X}e^{-r_c(T-t)}Se^{r_c(T-t)-\delta(T-t)}\right] \\
&= e^{-\delta(T-t)}N(d_1)
\end{aligned}$$

b. Partial derivative with regard to σ:

$$\begin{aligned}
\frac{\partial C}{\partial \sigma} &= Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial \sigma} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial \sigma} \\
&= Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \sigma} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial \sigma} - Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial \sigma} - Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_2}{\partial \sigma} \\
&= Se^{-\delta(T-t)}n(d_1)\left[\underbrace{\frac{\partial d_1}{\partial \sigma} - \frac{\partial d_2}{\partial \sigma}}_{\sqrt{T-t}}\right] \\
&= S\sqrt{T-t}\;n(d_1)e^{-\delta(T-t)}
\end{aligned}$$

c. Partial derivative with regard to (T-t):

$$-\frac{\partial C}{\partial(T-t)} = \delta\, Se^{-\delta(T-t)}N(d_1) - Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial(T-t)} - r_c Xe^{-r_c(T-t)}N(d_2) + Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial(T-t)}$$

$$= \delta\, Se^{-\delta(T-t)}N(d_1) - Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial(T-t)} - r_c Xe^{-r_c(T-t)}N(d_2) + Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial(T-t)}$$

$$= \delta\, Se^{-\delta(T-t)}N(d_1) - Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial(T-t)} - r_c Xe^{-r_c(T-t)}N(d_2) + Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial(T-t)}$$

$$= \delta\, Se^{-\delta(T-t)}N(d_1) - Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial(T-t)} - r_c Xe^{-r_c(T-t)}N(d_2) + Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_2}{\partial(T-t)}$$

$$= \delta\, Se^{-\delta(T-t)}N(d_1) - r_c Xe^{-r_c(T-t)}N(d_2) + Se^{-\delta(T-t)}n(d_1)\left[\underbrace{\frac{\partial d_1}{\partial(T-t)} - \frac{\partial d_2}{\partial(T-t)}}_{-\frac{\sigma}{2\sqrt{T-t}}}\right]$$

$$= -\frac{Sn(d_1)\sigma\, e^{-\delta(T-t)}}{2\sqrt{T-t}} - r_c Xe^{-r_c(T-t)}N(d_2) + \delta\, Se^{-\delta(T-t)}N(d_1)$$

d. Partial derivative with regard to r_c:

$$\frac{\partial C}{\partial r_c} = Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial r_c} + (T-t)Xe^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial r_c}$$

$$= Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial r_c} + (T-t)Xe^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial r_c}$$

$$= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial r_c} + (T-t)Xe^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial r_c}$$

$$= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial r_c} + (T-t)Xe^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_1}{\partial r_c}$$

$$= n(d_1)\frac{\partial d_1}{\partial r_c}\left[Se^{-\delta(T-t)} - \frac{X}{X}e^{-r_c(T-t)}Se^{r_c(T-t)-\delta(T-t)}\right] + (T-t)Xe^{-r_c(T-t)}N(d_2)$$

$$= X(T-t)e^{-r_c(T-t)}N(d_2)$$

e. Partial derivative with regard to X:

$$\frac{\partial C}{\partial X} = Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial X} - e^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial X}$$

$$= Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial X} - e^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial X}$$

$$= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial X} - e^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial X}$$

$$= Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial X} - e^{-r_c(T-t)}N(d_2) - Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_1}{\partial X}$$

$$= n(d_1)\frac{\partial d_1}{\partial X}\left[Se^{-\delta(T-t)} - \frac{X}{X}e^{-r_c(T-t)}Se^{r_c(T-t)-\delta(T-t)}\right] - e^{-r_c(T-t)}N(d_2)$$

$$= -e^{-r_c(T-t)}N(d_2)$$

f. Partial derivative with regard to δ:

$$\frac{\partial C}{\partial \delta} = -(T-t)Se^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial \delta} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial \delta}$$

$$= -(T-t)Se^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial \delta} - Xe^{-r_c(T-t)}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial \delta}$$

$$= -(T-t)Se^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial \delta} - Xe^{-r_c(T-t)}n(d_2)\frac{\partial d_2}{\partial \delta}$$

$$= -(T-t)Se^{-\delta(T-t)}N(d_1) + Se^{-\delta(T-t)}n(d_1)\frac{\partial d_1}{\partial \delta} - Xe^{-r_c(T-t)}n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}\frac{\partial d_1}{\partial \delta}$$

$$= -(T-t)Se^{-\delta(T-t)}N(d_1) + n(d_1)\frac{\partial d_1}{\partial \delta}\left[Se^{-\delta(T-t)} - \frac{X}{X}e^{-r_c(T-t)}Se^{r_c(T-t)-\delta(T-t)}\right]$$

$$= -S(T-t)e^{-\delta(T-t)}N(d_1)$$

g.Auxiliary relationships:

Auxiliary relationships 1:

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$d_2^2 = d_1^2 - 2d_1\sigma\sqrt{T-t} + \sigma^2(T-t)$$

$$= d_1^2 - 2\left[\ln\left(\frac{S}{X}\right) + \left(r_c - \delta + \frac{1}{2}\sigma^2\right)(T-t)\right] + \sigma^2(T-t)$$

$$= d_1^2 - 2\left[\ln\left(\frac{S}{X}\right) + (r_c - \delta)(T-t)\right] - \frac{2}{2}\sigma^2(T-t) + \sigma^2(T-t)$$

$$= d_1^2 - 2\ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right), \quad or$$

$$d_1^2 = d_2^2 + 2\ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right)$$

Auxiliary relationships 2:

$$N(d_1) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{d_1} \exp\left(-z^2/2\right)dz$$

$$\frac{\partial N(d_1)}{\partial S} = n(d_1)\frac{\partial d_1}{\partial S}$$

$$N(d_2) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{d_2} \exp\left(-z^2/2\right)dz$$

$$\frac{\partial N(d_2)}{\partial S} = n(d_2)\frac{\partial d_2}{\partial S}$$

Auxiliary relationships 3:

$$n(d_1) = \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_1^2}{2}}$$

$$= \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_2^2}{2} + \ln\left(\frac{X}{Se^{(r_c-\delta)(T-t)}}\right)}$$

$$= \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_2^2}{2}} e^{\ln\left(\frac{X}{Se^{(r_c-\delta)(T-t)}}\right)}$$

$$= n(d_2)\frac{X}{Se^{(r_c-\delta)(T-t)}}$$

$$n(d_2) = \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_2^2}{2}}$$

$$= \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_1^2}{2} + \ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right)}$$

$$= \frac{1}{\sqrt{2\Pi}} e^{-\frac{d_1^2}{2}} e^{\ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right)}$$

$$= n(d_1)\frac{Se^{(r_c-\delta)(T-t)}}{X}$$

Auxiliary relationships 4:

$$\frac{\partial d_1}{\partial \sigma} = -\frac{\ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right)}{\sigma^2\sqrt{T-t}} + \frac{1}{2}\sqrt{T-t}$$

$$\frac{\partial d_2}{\partial \sigma} = -\frac{\ln\left(\frac{Se^{(r_c-\delta)(T-t)}}{X}\right)}{\sigma^2\sqrt{T-t}} - \frac{1}{2}\sqrt{T-t}$$

$$\frac{\partial d_1}{\partial (T-t)} = -\frac{\ln\left(\frac{S}{X}\right)}{2\sigma (T-t)^{3/2}} + \frac{(r_c - \delta)}{2\sigma\sqrt{T-t}} + \frac{\sigma}{4\sqrt{T-t}}$$

$$\frac{\partial d_2}{\partial (T-t)} = -\frac{\ln\left(\frac{S}{X}\right)}{2\sigma (T-t)^{3/2}} + \frac{(r_c - \delta)}{2\sigma\sqrt{T-t}} - \frac{\sigma}{4\sqrt{T-t}}$$

$$\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$$

$$\frac{\partial d_1}{\partial r_c} = \frac{\partial d_2}{\partial r_c}$$

$$\frac{\partial d_1}{\partial \delta} = \frac{\partial d_2}{\partial \delta}$$